Real Time Reliability Adequacy Tools

Time-space Methods for Determining Locational Reserves: A Framework for Location-based Pricing and Scheduling for Reserve Markets

Final Project Report

December 2002

Research Team Faculty

James S. Thorp, Project Leader
School of Electrical & Computer Engineering
Cornell University

Robert J. Thomas
School of Electrical & Computer Engineering
Cornell University

Research Team Student

Jie Chen
School of Electrical & Computer Engineering
Cornell University

ACKNOWLEDGEMENTS

The work described in this report was coordinated by the Consortium for Electric Reliability Technology Solutions, and funded by the Assistant Secretary of Energy Efficiency and Renewable Energy, Office of Distributed Energy and Electricity Reliability, Transmission Reliability Program of the U.S. Department of Energy under Interagency Agreement No. DE-AI-99EE35075 with the National Science Foundation.

EXECUTIVE SUMMARY

It is well known that given a network that can become constrained on voltage or real power flows, reserves must also be spatially located in order to handle all credible contingencies. However, to date, there is no credible science-based method for assigning and pricing reserves in this way. Presented in this work is a new scheduling algorithm incorporating constraints imposed by grid security considerations, which include one base case (intact system) and a list of possible contingencies (line-out, unit-lost, and load-growth) of the system. By following a cost-minimizing co-optimization procedure, both power and reserve are allocated spatially for the combined energy and reserve markets. With the Lagrange multipliers (dual variables) obtained, the scheduling algorithm also reveals the locational shadow prices for the reserve and energy requirements. Unlike other pricing and scheduling methods for reserves in use, which are usually ad-hoc and are based on engineering judgment and experience, this proposed formulation is likely to perform better in restructured markets when market power is a potential problem. The modified IEEE 30-bus system is used to introduce concepts and present results.

TABLE OF CONTENTS

1. INTRODUCTION

1.1. Background of operating reserves

An overriding factor in the power system operation is the maintenance of system security. Historically, the term security, when applied to the electric power system, refers to the ability of the bulk system to withstand sudden disturbances such as electric short circuits or unanticipated loss of system components [1]. The static nature of the problem, that is, guaranteeing that in the post-contingency state all power system components are operating within established limits, is tractable once the set of credible contingencies is known. Generally, the most severe contingency is the sudden and unanticipated loss of a large generating unit although the loss of a critical line or a sudden and large increase in load at strategic locations could be just as catastrophic. The problem of whether or not the system can survive the transition, that is, the dynamic nature of system security, is still a hard and unresolved problem. Since in most systems load is not dispatchable, the security of the system depends on having the proper level, location and type of operating reserves available when needed to meet a contingency.

In the era of electricity regulation, vertically integrated utilities provided operating reserves through the advice of North American Electric Reliability Council (NERC) and regional reliability councils. Under deregulation, however, their procurement usually is the duty of the Independent System Operator (ISO). In the restructured system, reserves have both an engineering role and an economic role. The engineering role is to ensure that load is met in an environment where there is a regulatory obligation to serve load. The economic role of reserves is to avoid the losses associated with outages. The need for reserves is exacerbated by the fact that load usually is price inelastic. That is, there is an obligation to serve demand regardless of its level or location. Because of the network and the constraints it imposes, load may be isolated from generation if reserves are not placed properly with respect to a contingency.

All generators have ramp rate constraints that must be taken into account when assigning reserves. These are constraints on how fast a unit can change its output. Generally a unit's ramp rate is about one percent of its capacity per minute. So, if a unit has ramping capability (that is, it has the ancillary systems necessary to control the unit set-point) and its capacity is 100MW, it can be expected to supply about 1MW per minute. Operating reserves are often classified into four categories: 1) Regulation for Automatic Generation Control (AGC for load following), 2) 10-

minute spinning reserve that is usually supplied by generators operating at less than full capacity. A unit with a 4MW/min ramp rate can supply 40MW's of spinning reserves, 3) 10-minute non-spinning reserves that can be supplied by off-line generation that can be started quickly, and 4) 30 to 60-minute non-spinning reserves that can be supplied by off-line generation that can be started and ramped in that time frame. Spinning reserve normally should be no less than one-half the operating reserves required for each settlement period of the market.

1.2. Existing reserve market – fixed reserve requirement

Although the focus of deregulation has been on the design of markets for the efficient delivery of electricity, the role of reserves in maintaining the safe and reliable system operation is equally critical to the market performance. And a well functioning reserve market can also help mitigate price spikes and solve the capacity problem. Currently there are markets for energy, and markets for reserves exist in some form in most currently operating ISOs. Also, a specific form of reserve market is proposed in the Standard Market Design Notice of Proposed Rulemaking (SMD NOPR) issued recently by the Federal Energy Regulatory Commission (FERC). Although there are different ways of procuring reserves, a common feature does exist: deterministic reserve requirement, which ensure that the reserve is sufficient to make up for the loss of the largest unit or that the reserve must be a given percentage of forecasted peak demand or some combination of these. Table 1 shows an example of fixed reserve requirement currently in use in New York ISO. There are three different requirements for three zones, and the assignment is based on the most severe NYCA (New York Control Area) operating capability loss (1200MW).

Currently, reserves are only thought of as having time dependent properties, that is, they must be spinning or able to synchronize in ten minutes or within other predefined timeframes. However, it is well known that given a network that can become constrained on voltage or real power flows, reserves must also be spatially located in order to handle all the contingencies that could occur. To date, there is no credible science-based method for assigning reserves in this way. Virtually all methods are ad-hoc and are based on engineering judgment and experience.

Table 1. Fixed reserve requirement in NYISO electricity market

	New York CA	Eastern New York	Long Island
	A = most severe NYCA operating capability loss (1200MW)		
10 Minute Spinning Reserve	½ A = 600MW (I)	1/4 A = 300MW (IV)	1/20 A = 60MW (VII)
10 Minute Total Reserve	A = 1200MW (II)	1200MW (V)	1/10 A = 120MW (VIII)
30 Minute Reserve	1½ A = 1800MW (III)	1200MW (VI)	270-540MW (IX)

The fixed reserve requirement works well under regulation, because all generators in a given area cooperate to configure the flows to maintain the system in operation. In a deregulated market, however, no cooperation exists. One potential problem with the fixed reserve requirement is that the reserve procurement may not be locationally distributed as desired. The consequence could be that some contingencies, if occur, may not be covered with the procured reserves although should have to (not because of shortfall in quantity, but due to the allocations in the wrong place), resulting in increased operating cost - expensive imports are needed. To avoid this situation or at least try to minimize the chance of happening, one possible way is to determine a big number (reserve requirement in MWs) so that more generators will have chance to carry reserves. The reserve assignment then might be closer to the desired spatial allocation – be able to make up every planned contingency. But the big number often means wasting resources. The system would require less reserves if they can be locationally assigned in a “smart” way.

Since a generator is a multi-commodity device, that is, it can supply energy, reserves and VARs all at the same time, payments should be made for each commodity it provides to the system. Under a restructured system, markets should determine the fair price for each commodity. In the arena of deregulated electric power markets, the concept of location-based marginal pricing (LBMP) is well-established and is commonly used to set electricity prices at a nodal level, making it possible to assign the cost of congestion to the locations that create it in the first place. However, in the reserve markets, no standard methods for location-based pricing of reserves exist. Most of the ISOs are pricing reserves uniformly. Part of the problem lies in the fact that reserves procured in the market are based on the deterministic reserve requirement, which is similar to the energy dispatch based on the Economic Dispatch (ED) with no consideration of underlying transmission networks, in which only one electricity price for all nodes is obtained.

1.3. Alternative market proposal – variable/responsive reserve requirement

This project explored a different way to allocate and price reserves. The requirement here is that the reserves procured in this way will maintain the same level of security for the transmission system, i.e., be able to cover the same set of contingencies considered in the fixed reserve market. It is similar to the method used in [3-4], in which system security is evaluated using probability-weighted performance indices over a set of power-flow cases or a set of credible contingencies. Therefore, there is no need to have a fixed reserve requirement included in the optimization. Instead, the same set of contingencies resulting in the deterministic reserve requirement will be included in the proposed scheduling and pricing algorithm. Locational assignments and locational prices for energy and reserves are available through the algorithm and are based on a "true" co-optimization of both energy and reserves. The actual amount of reserves assigned varies with different system demands and energy-reserve offers. Hence, we call this new co-optimization Responsive Reserves (RR) to distinguish it from the conventional form of Fixed Reserves (FR).

2. UNDERLYING OPTIMIZATION FRAMEWORK

2.1. Notation

In this report the following notation will be used. Additional symbols will be introduced when necessary.

i:	generator index ($i = 1, 2, \cdots, I$)
j:	bus index ($j = 1, 2, \cdots, J$)
l:	transmission line index ($l = 1, 2, \cdots, L$)
k:	contingency index ($k = 0, 1, \cdots, K$), 0 indicates the base case (intact system), predefined contingencies otherwise.
P_{ik} / Q_{ik}:	real/reactive power output of generator i in the kth contingency.
R_{ik}:	spinning reserve carried by generator i in the kth contingency.
q_{jk}:	voltage angle of bus j in the kth contingency.
V_{jk}:	voltage magnitude of bus j in the kth contingency.
S_{lk}:	power flow of line l in the kth contingency.
$P_i^{\min}, P_i^{\max}$:	minimum and maximum real power capacity for generator i
$Q_i^{\min}, Q_i^{\max}$:	minimum and maximum reactive power capacity for generator i
$R_i^{\max}$:	maximum reserve for generator i
$V_j^{\min}, V_j^{\max}$:	voltage magnitude limits for bus j
$S_l^{\max}$:	power flow limit for line l
$C_{P_i}(P_{ik})$:	energy cost for operating generator i at output level P_{ik} in the kth contingency.
$C_{R_i}(R_{ik})$:	reserve cost for generator i carrying R_{ik} spinning reserve in the kth contingency.
p_k:	the probability of the kth contingency

2.2. Co-optimization (CO-OPT) formulation

In order to set up the formulation, a few assumptions are made below for the integrated energy and spinning reserve market:

1) The energy and reserves are settled simultaneously based on a true co-optimization.
2) Only 10-min spinning reserves are considered.
3) No double auction is considered for simplicity. This means the demand side is fixed, price-inelastic, and energy suppliers are the only market participants.
4) Inter-temporal constraints are ignored, and schedules of different trading periods are independently determined.
5) The CO-OPT considers a base-case system - intact system that runs smoothly with no failures, and a set of specified contingencies, which may contain line-out, unit failure, or unexpected load growth. These cases are predefined, and only one case happens at a time.
6) A set of probabilities which are assigned to the base case and listed contingencies are also known.
7) The fully AC model is used to accommodate physical network limits.

These assumptions are intended to conceptually elaborate the proposed scheduling algorithm; however, the algorithm and the solution are not necessarily limited by them.

The ISO requires an optimization procedure to determine the schedules to every supplier. The objective here is to minimize the total expected cost (operating energy cost plus the spinning reserve cost) over the predefined base case and credible contingencies, stated as follows,

$$\min_{P,\,R} \quad \sum_{k=0}^{K} p_k \left\{ \sum_{i=1}^{I} \left[C_{P_i}(P_{ik}) + C_{R_i}(R_{ik}) \right] \right\} \tag{1}$$

The minimization is subject to network and system constraints enforced by each of the base case and contingencies. These constraints include nodal power balancing constraints,

$$F_{jk}(q, V, P, Q) = 0\,, \qquad j = 1, \cdots, J \qquad k = 0, \cdots, K \tag{2}$$

line power flow constraints (detailed formulations for (2) and (3) are referred to [5]),

$$\left| S_{lk} \right| \le S_l^{\max}\,, \qquad l = 1, \cdots, L \qquad k = 0, \cdots, K \tag{3}$$

voltage limits

$$V_j^{\min} \le V_{jk} \le V_j^{\max}, \quad j = 1, \cdots, J \quad k = 0, \cdots, K \tag{4}$$

real power limits

$$P_i^{\min} \le P_{ik} \le P_i^{\max}, \quad i = 1, \cdots, I \quad k = 0, \cdots, K \tag{5}$$

reactive power limits

$$Q_i^{\min} \le Q_{ik} \le Q_i^{\max}, \quad i = 1, \cdots, I \quad k = 0, \cdots, K \tag{6}$$

spinning reserve ramping limits

$$0 \le R_{ik} \le R_i^{\max}, \quad i = 1, \cdots, I \quad k = 0, \cdots, K \tag{7}$$

and unit capacity limits

$$P_{ik} + R_{ik} \le P_i^{\max}, \quad i = 1, \cdots, I \quad k = 0, \cdots, K \tag{8}$$

Notice that in (5) ~ (8), $P_i^{\max}$ and $R_i^{\max}$ are from the submitted offers, which may be lower than the actual physical limits due to sellers' intentionally withholding of capacity.

The formulation so far can be decoupled into K+1 separate sub-problems (corresponding to specified K+1 systems) unless the concept of *Total Unit Committed Capacity (TUCC)* is introduced to tie them up. The *TUCC* of unit i in the k[th] contingency is defined as

$$G_{ik} = P_{ik} + R_{ik}, \quad i = 1, \cdots, I \quad k = 0, \cdots, K \tag{9}$$

If a contingency such as a line-out or a unit failure occurs, the common remedy will be to fix the problem as soon as possible and bring the power grid back to its normal operation condition (the base case). Hence, units are also expected to return to the base case dispatches (least cost solution) upon the return of the failed component. To make this remedy possible for every listed contingency case, the *TUCC* required in each of the contingencies should be more than or at least equal to the base case *TUCC*. Meanwhile since our goal here is minimize the total cost, we want as little capacity committed into the market as possible while still meeting the security criteria.

For this purpose, the *TUCC* for any generator i is required to be the same over all K+1 cases, that is,

$$G_{ik_1} = G_{ik_2}, \qquad i = 1, \cdots, I \quad k_1, k_2 = 0, \cdots, K \tag{10}$$

From (9) and (10), R_{ik} can be written as

$$R_{ik} = R_{i0} + P_{i0} - P_{ik}, \quad i = 1, \cdots, I \quad k = 1, \cdots, K \tag{11}$$

The equality constraints (11) then tie up the whole problem. Meanwhile, in the implementation, we can keep the base case reserve decision variables $R_{i0} (i = 1, \cdots, I)$ only and get rid of all other reserve decision variables by substituting the right hand side of (11) for wherever $R_{ik} (i = 1, \cdots, I; k = 1, \cdots, K)$ is used. By doing so, the problem size can be reduced such that implementation efficiency is improved. However, for the ease of conceptual illustration, we keep all R_{ik}.

2.3. Solution properties

$P_i^{\min}, P_i^{\max}$ and $R_i^{\max}$ are the physical limits for unit i. They define the outer box (black dotted) in Figure 1, together with the 45-degree line indicating the unit capacity limit constraint,

$$P_i + R_i \le P_i^{\max} \tag{12}$$

The region inside the box is the feasible operating region for unit i. But, usually participating units will make strategic offers by withholding capacity according to real-time market situations. The offered-in limits $\tilde{P}_i^{\max}$ and $\tilde{R}_i^{\max}$ ($P_i^{\min} \le \tilde{P}_i^{\max} \le P_i^{\max}$, $0 \le \tilde{R}_i^{\max} \le R_i^{\max}$) thus define a smaller feasible operating region (inner blue dashed box), within which the optimal dispatch for unit i is scheduled.

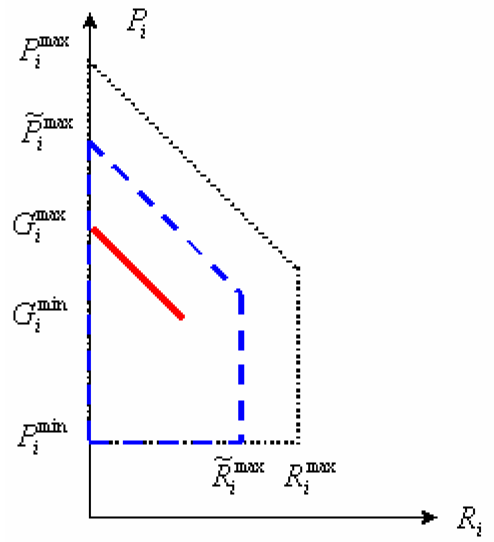


Figure 1. Offer and solution pattern

The co-optimization contains (K+1) Optimal Power Flows (OPFs) only coupled by the reserve costs and the dependence of reserves on generations. Generally the optimal solution is different than (K+1) separate OPF's that do not consider the reserves. Assume the optimal energy dispatch for all K+1 cases, expressed in matrix, is

$$\mathbf{P} = \begin{bmatrix} P_{10} & P_{20} & \cdots & P_{I0} \\ P_{11} & P_{21} & \cdots & P_{I1} \\ \vdots & \vdots & \vdots & \vdots \\ P_{1K} & P_{2K} & \cdots & P_{IK} \end{bmatrix} \tag{13}$$

Likewise, the optimal reserve allocation is

$$\mathbf{R} = \begin{bmatrix} R_{10} & R_{20} & \cdots & R_{I0} \\ R_{11} & R_{21} & \cdots & R_{I1} \\ \vdots & \vdots & \vdots & \vdots \\ R_{1K} & R_{2K} & \cdots & R_{IK} \end{bmatrix} \tag{14}$$

Let

$$\begin{aligned} G_i^{\min} &= \min(P_{i0}, P_{i1}, \cdots, P_{iK}) \\ G_i^{\max} &= \max(P_{i0}, P_{i1}, \cdots, P_{iK}) \qquad i = 1, \cdots, I \end{aligned} \tag{15}$$

In the optimal dispatch, for any unit i, there exists at least one case (out of K+1 cases), its *TUCC* is consumed as energy only, that is, for that particular case (usually is the "largest" system contingency for unit i), unit i does not carry any spinning reserve. That is, all the spinning reserves carried by unit i pick up part of the unserved load due to failure of other components. So, $G_i^{\max}$ is unit i's *TUCC*

$$G_{ik} = G_i^{\max}, \qquad i = 1, \cdots, I \quad k = 0, \cdots, K \tag{16}$$

So, by performing the co-optimization, the ISO will assign every participating unit a commitment interval $[G_i^{\min}, G_i^{\max}]$. $G_i^{\min}$ is the minimum energy output required from unit i for the real-time market, additional energy within that interval may or may not be scheduled according to real-time system situation. The residual committed capacity will still be available and paid as reserves. The actual real-time operating point is thus along the red solid line in Figure 1 and depends on the real-time system condition.

2.4. Augmented Optimal Power Flow (AOPF)

CO-OPT determines the optimum energy dispatch and reserve allocation for all the K+1 cases. Since the objective is to minimize the expected costs over all K+1 cases, the obtained energy and reserve shadow prices are also in such an "expected" fashion. However, suppliers would expect to be paid in a real-time, state-dependent fashion, i.e., the payment will depend on the actual real-time system condition. This requires a single OPF-like optimization to be solved in real-time not only producing the same dispatches as in co-optimization solutions but also revealing spot nodal prices. The Augmented OPF (AOPF), which adds reserves to the traditional OPF, is introduced below to do the job.

The AOPF is defined as the sub-problem of the co-optimization, which is the cost-minimizing optimization for one of the specified K+1 systems (base case or contingencies). The objective for the kth AOPF is to minimize the total energy and reserve cost of the kth case.

$$f_k = \min_{P,\, R} \sum_{i=1}^{I} \left[C_{P_i}(P_{ik}) + C_{R_i}(R_{ik}) \right] \tag{17}$$

The constraints defined for the k^th system in (2) ~ (7) still hold, and the only difference is that the generation limits $(P_i^{\min}, P_i^{\max})$ are replaced by committed intervals $(G_i^{\min}, G_i^{\max})$ carried on from the CO-OPT. In particular, generation limits in (5) are rewritten as

$$G_i^{\min} \le P_{ik} \le G_i^{\max} \tag{18}$$

And the available spinning reserve is defined as

$$R_{ik} = G_i^{\max} - P_{ik} \tag{19}$$

The AOPF has the required property as shown by the following proposition.

Proposition 1 If P (13) and R (14) are the optimal solutions to the CO-OPT (1), then for any $k \in \{0,1,\cdots,K\}$, $\overline{P}_k =$ P(k,:) and $\overline{R}_k =$ R(k,:) are also the solutions to the k^th AOPF (17~19).

Proof. If not, then there exists at least one k $(0 \le k \le K)$, such that $(\hat{\overline{P}}_k, \hat{\overline{R}}_k)$ is the optimal solution to the k^th AOPF, but $\hat{\overline{P}}_k \ne \overline{P}_k$ and $\hat{\overline{R}}_k \ne \overline{R}_k$. Since $(\hat{\overline{P}}_k, \hat{\overline{R}}_k)$ produces lower cost to the k^th AOPF than $(\overline{P}_k, \overline{R}_k)$ does, substituting $(\overline{P}_k, \overline{R}_k)$ with $(\hat{\overline{P}}_k, \hat{\overline{R}}_k)$ in the optimal solution $(\mathbf{P}, \mathbf{R})$ to (1) should not only form a feasible solution, but also produce lower total expected cost, contradicting the fact that $(\mathbf{P}, \mathbf{R})$ is the optimal solution. QED

3. LOCATION-BASED PRICING

3.1. Competitive price and marginal cost

Economic theory suggests that price be equal to marginal cost in a competitive market. And quote about the definition of marginal cost from stoft[6] says,

The MIT Dictionary of Modern Economics (1992) defines marginal cost as "the extra cost of producing an extra unit of output". Paul Samuelson (1973) defines marginal cost more cautiously as "the cost of producing one extra unit more (or less)".

The definition assumes that the cost of producing one more unit of output (Incremental Cost) would be exactly as much as the saving of producing one less unit (Decremental Cost). This is true for the continuous cost curves but not for the discontinuous cost curves. The reason is as follows.

Mathematically, the marginal cost of generator i is defined as the ***derivative*** of its cost curve [spot pricing book],

$$l_i = \frac{\partial C_{P_i}(P_i)}{\partial P_i} \tag{20}$$

But at the discontinuous points, if any, of the cost curve, the definition (20) does not exist. However, two other derivatives (which are also defined for continuous points) are defined:

left-hand-side derivative

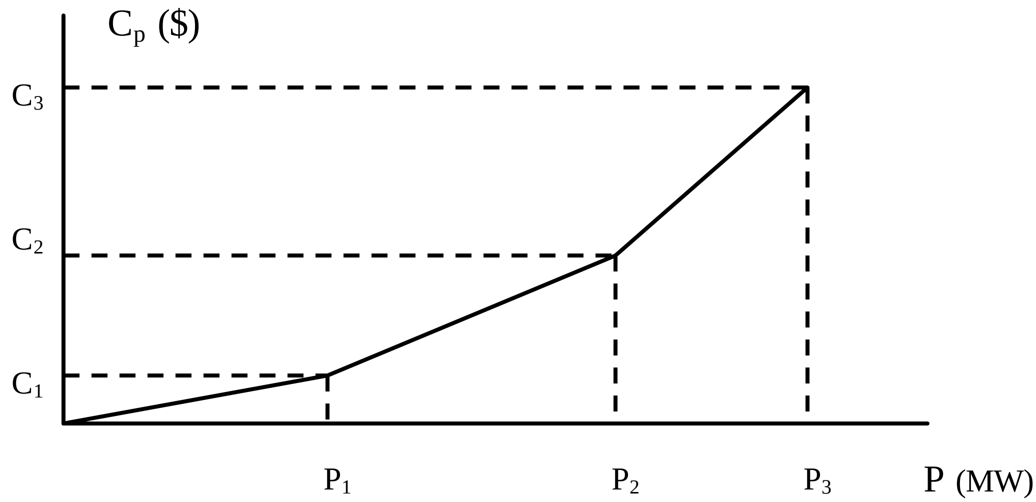
$$l_i^- = \frac{\partial C_{P_i}(P_i^-)}{\partial P_i^-} \tag{21}$$

and ***right-hand-side derivative***

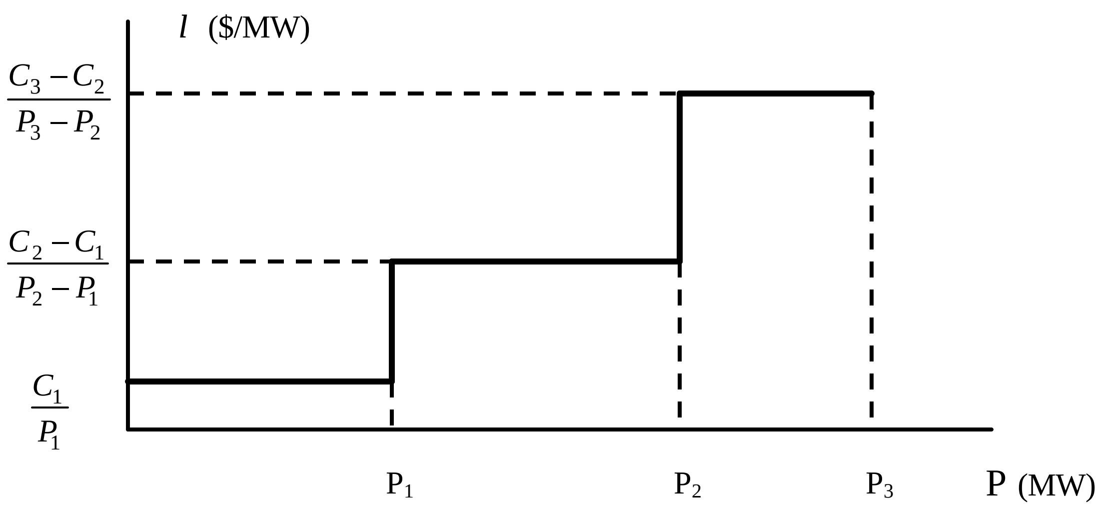
$$l_i^+ = \frac{\partial C_{P_i}(P_i^+)}{\partial P_i^+} \tag{22}$$

Equ. (21) and (22) are often interpreted as unit i's decremental and incremental costs at the dispatch point of P_i. The relationship between these derivatives (or marginal costs) and the competitive price in the electricity market is of special interest to us.

Multi-block price-quantity bids are common in the electricity market, which can be represented by piecewise linear cost curves. Figure 2 shows an example of a three-block energy offer and its derivative.



(a) piecewise linear cost curve



(b) derivative of the cost curve

Figure 2. Example of a 3-block energy offer

Clearly, the cost curve is discontinuous. All three derivatives (20~22) are well defined at those non-break points and they have the relationship of

$$l_i^- = l_i = l_i^+ \qquad (23)$$

However, at those break points (0, P_1, P_2, or P_3), (20) is not defined. And since the cost curve is non-decreasing, i.e., convex, the decremental cost is less than the incremental cost

$$l_i^- < l_i^+ \qquad (24)$$

So, for every point, (25) holds

$$l_i^- \le l_i^+ \qquad (25)$$

Assume there are I units participating in the market, and for simplicity no underlying physical transmission network is considered. Then (26) holds at the system optimum dispatch point

$$l_i^-(P_i) \le l_j^+(P_j) \qquad \forall i \in (1, I) \qquad \forall j \in (1, I) \qquad (26)$$

Proof. If not, then there exists at least one pair of units m and n ($m, n \in (1, I)$) such that $l_m^-(P_m) > l_n^+(P_n)$. In this case, the solution of unit m producing one less unit and unit n producing one more unit will have lower cost, contradicting (P_m, P_n) is the optimum dispatch for both units. QED

Hence, we can safely define the Market's Incremental Cost (MIC), i.e., the least cost for the market to produce one more unit of output, as

$$MIC = \min(l_i^+, i = 1, 2, \cdots, I) \qquad (27)$$

and similarly, define the Market's Decremental Cost (MDC), i.e., the most savings for the market to produce one less unit of output, as

$$MDC = \max(l_i^-, i = 1, 2, \cdots, I) \qquad (28)$$

Following (26), we have

$$MDC \le MIC \tag{29}$$

The competitive market price (CMP) thus should be within the range of MDC and MIC[6].

$$MDC \le CMP \le MIC \tag{30}$$

3.2. Principle of location-based marginal pricing (LBMP)

The economic rationale for applying marginal cost pricing to an electricity network using the concepts of LBMP was presented in [9]. LBMP in electricity recognizes that the marginal price may differ at different locations and times. Differences result from transmission congestion and transmission losses. LBMP is believed to be an efficient market-based method for transmission congestion control and was also recommended in the SMD NOPR. Currently, LBMP is being used in the PJM and NYISO energy markets. And other ISOs, like ISO-New England and CAISO, also propose to adopt LBMP in the near future.

Although the derivation and discussion in section 4.1 are based on the uniform pricing without consideration of the transmission network, the results are still valid in the new context of LBMP but at a nodal level. According to [9], the locational marginal cost for energy is quantified as

$$l_k(t) = \frac{\partial \ \text{[Total energy cost]}}{\partial \, d_k(t)} \tag{31}$$

where $l_k(t)$ is the energy marginal cost for the kth node at time t, and $d_k(t)$ is load demand at the kth node at time t. The evaluation of (31) is subject to various network constraints.

Applying the same logic as in section 4.1 here, we can define Nodal Incremental Cost (NIC) and Nodal Decremental Cost (NDC) in a similar fashion,

$$NIC_k(t) = \frac{\partial \ [\text{Total energy cost}^{+}]}{\partial \, d_k(t)^{+}} \tag{32}$$

$$NDC_k(t) = \frac{\partial \ [\text{Total energy cost}^{-}]}{\partial \, d_k(t)^{-}} \tag{33}$$

Similarly, when (31) is well defined at the optimum dispatch point, $l_k(t)$ is the Nodal Energy Price, $NEP_k(t)$, and

$$NEP_k(t) = NDC_k(t) = l_k(t) = NIC_k(t) \tag{34}$$

However, when the optimum dispatch is located at some “special corners” of the feasible region (similar to the discontinuous points on the cost curve in Figure 2), the definition of (31) breaks down. In this case,

$$NDC_k(t) \le NEP_k(t) \le NIC_k(t) \tag{35}$$

In currently widely used optimization techniques, such as Lagrange Relaxation (LR), $l_k(t)$ is actually so called *Lagrange Multiplier(LM) (or Shadow Price (SP)* in the economics perspective) corresponding to the k^{th} nodal energy balance equation at time t. And because the characteristics of numerical algorithms, the SP can be obtained even if $l_k(t)$ (31) is not well defined at the solution point. Shown below is a simple example to illustrate the case.

Example: Assume there are two generators supplying total 4MW load. Each generator has capacity of 4MW. The cost for one generator is $1/MWh, $4/MWh for the other. Solve for the optimum dispatch and the market clearing price.

A simple linear program suffices to solve the problem. And the primal-dual method is used to get the shadow price[1], as shown in Table 2. The dual variable y_1 is the corresponding energy shadow price (SP), i.e., market clearing price. In this case, analytically and theoretically, the SP can be any number in the interval [1, 4]. That means the SP has multiple choices, i.e., the dual problem has multiple solutions. Notice that, 1 is the decremental cost (DC) and 4 is the incremental cost (IC) for the primal problem, so this verifies

$$DC \le SP \le IC \tag{36}$$

[1] The LR technique is based on the theory of duality [linear programming book]. The LMs obtained from the LR method are actually Dual Variables (the solutions of the dual problem) in the primal-dual formulation.

Table 2. An illustrative example

	PRIMAL PROBLEM	DUAL PROBLEM
Problem formulation	$\min\ x_1 + 4x_2$ s.t. $x_1 + x_2 = 4$ $x_1 \le 4,\quad x_2 \le 4$ $x_1 \ge 0,\quad x_2 \ge 0$	$\max\ 4y_1 - 4y_2 - 4y_3$ s.t. $y_1 - y_2 \le 1$ $y_1 + y_3 \le 4$ $y_2 \ge 0,\quad y_3 \ge 0$
Optimum solution	$x_1 = 4,\quad x_2 = 0$	$1 \le y_1 \le 4,\ y_2 = y_1 - 1,\ y_3 = 0$

As shown in Figure 3, the optimum solution is located at a "special corner", $l_k(t)$ at that point has no definition. While by using numerical techniques, the SP can be obtained. However, different numerical algorithms or even different settings for the starting point and step size in one algorithm might produce different SPs. For instance, for this little example, Matlab produces SP = **1.8634** by using **LINPROG** with *'Large-Scale Algorithm: lipsol'* chosen, but SP = **1.0** by choosing '*Medium-Scale Algorithm: activeset*'.

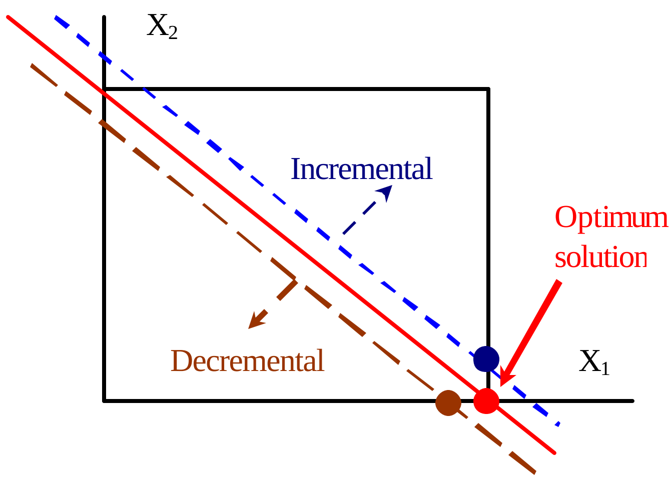


Figure 3. Graphic illustration of the example

Hence, only when both the primal and dual problems have unique solutions, the dual variables (or Lagrange multipliers) are correct shadow prices. When the dual problem has multiple solutions, there exists certain randomness in the obtained shadow prices by various numerical algorithms, which creates ambiguous market prices if used.

3.3. LBMP for the RR market

To avoid ambiguous market prices under some special market conditions as discussed in previous sections, the competitive LBMP can be explicitly based on decremental cost or incremental cost in order to convey CLEAR price signals to market participants. Herein, we use incremental cost as the basis.

In traditional OPF, the nodal energy prices as defined in (32) can be numerically calculated following below steps (assume the energy price at bus j is what we are after):

1. Do the original OPF, record the optimum operating cost as f_0.
2. Perturb the system by adding an extra unit of load at bus j.
3. Do the perturbed OPF, record the minimum post-perturbation operating cost as f_1.
4. The difference of f_1-f_0 then is the wanted nodal energy price.

In the RR market, nodal energy and reserve prices can be found in a similar way, but the perturbation is a bit subtle. Since we rest on the CO-OPT for the energy and reserve scheduling, the redispatch after perturbation in the AOPF should be consistent with the corresponding perturbed CO-OPT solution. From proposition 1, we know the guarantee here is that both AOPF and CO-OPT have the same committed intervals for each generator. So, in order to get energy prices, the perturbation has to be done to both CO-OPT and AOPF. In particular, the numerical calculation is performed as follows (assume again we are after the energy price at bus j):

1. Do the original co-optimization, carrying solved $[G_i^{\min}, G_i^{\max}]$ for every unit to the AOPF; Do the AOPF, record the optimum cost as f_0.
2. Perturb the co-optimization by adding one extra unit of load at bus j for each of the K+1 systems.
3. Do the perturbed co-optimization, finding out the new committed interval $[newG_i^{\min}, newG_i^{\max}]$ for every unit.
4. Perturb the AOPF by adding one extra unit of load at bus j.
5. Do the perturbed AOPF with $[newG_i^{\min}, newG_i^{\max}]$ enforced, record the optimum post-perturbation operating cost as f_1.
6. The difference of f_1-f_0 then is the wanted nodal energy price.

Similar procedure can be used to reveal the nodal reserve prices. Steps 1 ~ 3 are the same as above, but instead of doing perturbed AOPF, we do the un-perturbed AOPF but with

$[newG_i^{\min}, newG_i^{\max}]$ enforced such that *the* one extra unit of generation prepared in the CO-OPT stage becomes one extra unit of reserve for bus j in the AOPF, thus the cost difference is equal to the nodal reserve price at bus j. Detailed procedures are as follows:

1. Do the original co-optimization, carrying solved $[G_i^{\min}, G_i^{\max}]$ for every unit to the AOPF; Do the AOPF, record the optimum cost as f_0.
2. Perturb the co-optimization by adding one extra unit of load at bus j for each of the K+1 systems.
3. Do the perturbed co-optimization, finding out the new committed interval $[newG_i^{\min}, newG_i^{\max}]$ for every unit.
4. Do the un-perturbed AOPF with $[newG_i^{\min}, newG_i^{\max}]$ enforced, record the optimum post-perturbation operating cost as f_1.
5. The difference of f_1-f_0 then is the wanted nodal reserve price.

Although above procedures help make price signals clear, the numerical perturbation is very time-consuming to implement, especially for large-scale systems. Hence, we have to make trade-off between computation time and unambiguous prices. In practice, post-optimization sensitivity analysis can provide a much more efficient way to handle these prices, but it has Lagrange multipliers involved.

Assume l_j is the Lagrange multiplier associated with nodal real power balancing at bus j from the AOPF; $m_{G_i^{\min}}$ and $m_{G_i^{\max}}$ ($i = 1,2,\cdots,I$) are the Lagrange multipliers related to the upper and lower boundaries of the unit committed intervals from the AOPF. Define

$$a_{ij} = \frac{\Delta G_i^{\min}}{\Delta D_j} \tag{37}$$

$$b_{ij} = \frac{\Delta G_i^{\max}}{\Delta D_j} \tag{38}$$

where D_j is the real load at bus j. a_{ij} is the sensitivity of change of $G_i^{\min}$ with respect to the change of bus j load, that is, if there is one unit of load variation at bus j, a_{ij} indicates the corresponding shift of $G_i^{\min}$. b_{ij} has similar definition for $G_i^{\max}$. The real-time nodal energy price at bus j, $\overline{l}_j$, then can be calculated as

$$\overline{l}_j = l_j + \sum_{i=1}^{I}(a_{ij} m_{G_i^{\min}} + b_{ij} m_{G_i^{\max}}), \quad j = 1, \cdots, J \tag{39}$$

The real-time nodal reserve price at bus j, $\overline{m}_j$, is formulated as

$$\overline{m}_j = \sum_{i=1}^{I}(a_{ij} m_{G_i^{\min}} + b_{ij} m_{G_i^{\max}}), \quad j = 1, \cdots, J \tag{40}$$

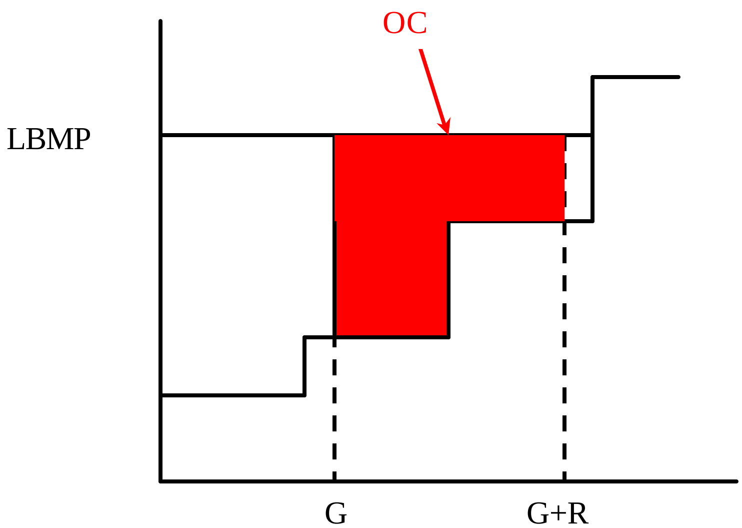
Therefore,

$$\overline{l}_j - \overline{m}_j = l_j \tag{41}$$

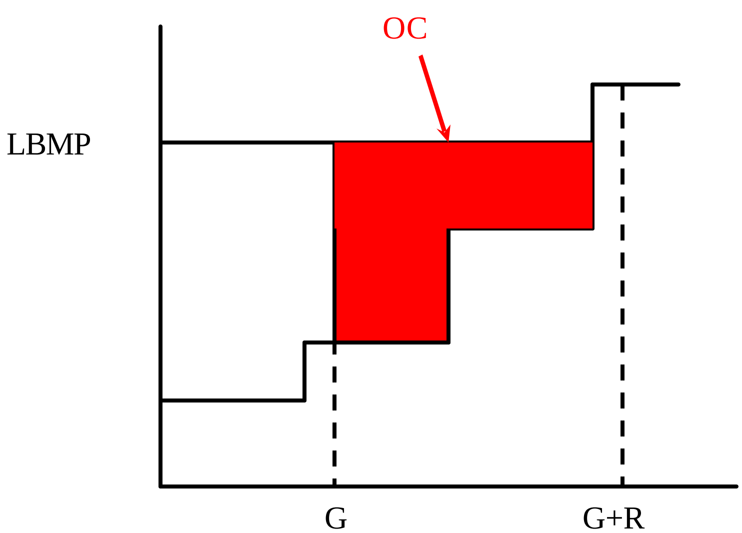
The interpretation of these calculations can still be put in the context of load perturbation. l_j will reflect the cost change in the AOPF if the load perturbation is done at bus j. Since the perturbation is performed in the AOPF without changing $[G_i^{\min}, G_i^{\max}]$ intervals, one unit of reserve will be called on to pick up the load perturbation, that is, one unit of reserve becomes one unit of energy. Therefore, the cost change involves both energy incremental cost and reserve decremental cost. That explains (41). And also recall that the reserve price can be obtained by doing the unperturbed AOPF with $[newG_i^{\min}, newG_i^{\max}]$. That means the change of $[G_i^{\min}, G_i^{\max}]$ actually affects the reserve allocation and hence its price, which is consistent with the formulation of (40). The numerical check of (39) ~ (41) can also be done by the above-described perturbation procedures.

3.4. Opportunity cost for reserves

The Opportunity Cost (OC) for a generator is the foregone profit associated with the provision of reserves, which is equal to the product of: (1) the quantity of reserves provided and (2) the price difference between (a) the LBMP existing at the time the generator was instructed to provide reserves and (b) the generator's energy offer for the same MW segment. Figure 4 further illustrates how the OC is calculated. Payment of OC for the reserves actually makes it indifferent, in the sense of profit-making, for the seller to supply energy or reserves, thus encouraging electricity suppliers to offer enough reserve capacity into the market.



(a) Energy offer (at G+R) <= LBMP



(b) Energy offer (at G+R) > LBMP

Figure 4. Illustration of OC computation

4. TEST SYSTEM

The test system being used for the proposed RR market is a modified IEEE 30-bus system shown in Figure 5.

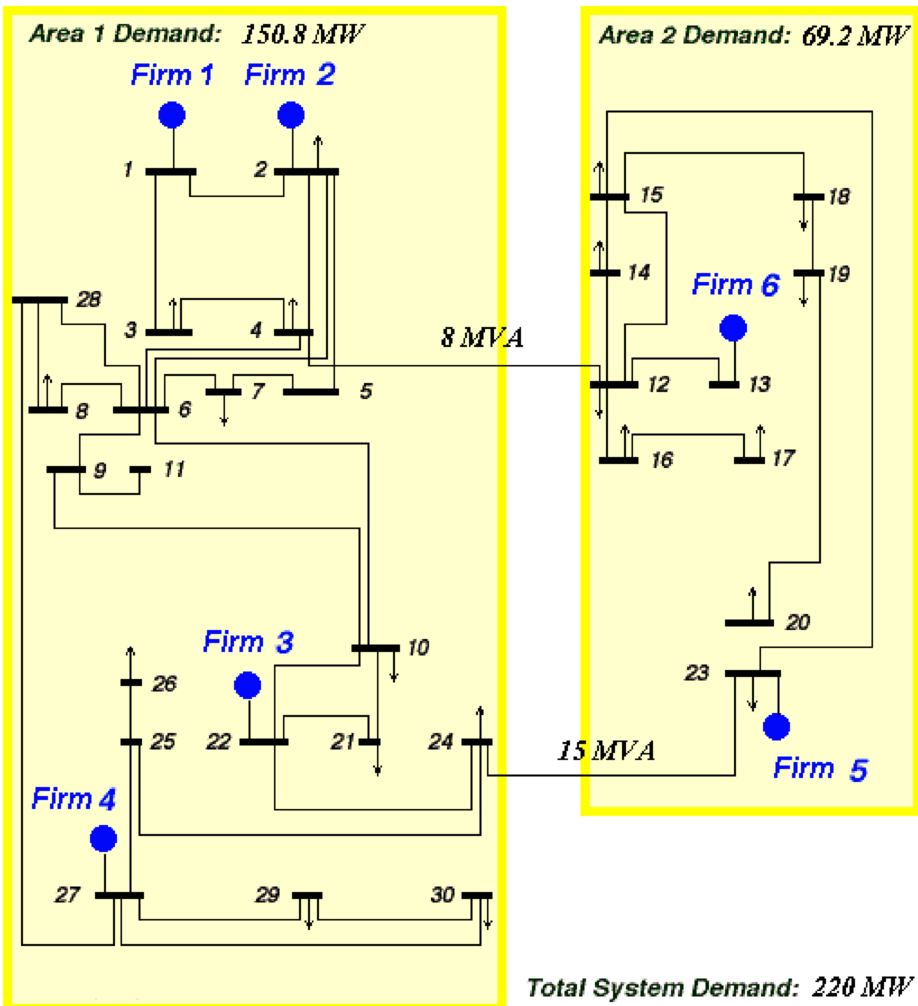


Figure 5. Modified IEEE 30-bus test system

There are six firms in the joint market run by the ISO. Firm 1,2,3 and 4 are located in area 1 while firm 5 and 6 are located in area 2. The transmission capacity between area 1 and area 2 is relatively limited (only 23 MVA in this case) compared to the transmission capacity within the two areas. Each firm owns two generators with a combined maximum capacity of 60 MW. The first generator has a maximum capacity of 40 MW, and the second has that of 20 MW. The two generators of each firm are the same within each area but different between areas. Table 3 lists generator data for firms in both areas. The system is designed so that the tie-lines between areas are usually congested making area 2 a load pocket, in which market power is easy to explore and excise. Interesting problems, such as the effects of transmission constraints and market power mitigation, therefore can be studied using this test system. (please see appendix A for detailed system data, and another real-size system model (see Appendix B) has already been developed and is ready for serious tests)

Table 3. Generator data

	AREA 1 FIRMS (1,2,3,4)		AREA 2 FIRMS (5, 6)	
	Gen #1	Gen #2	Gen #1	Gen #2
P_i^{min} (MW)	8.0	4.0	8.0	4.0
P_i^{max} (MW)	40.0	20.0	40.0	20.0
R_i^{max} (MW)	5.0	10.0	20.0	16.0
Energy Variable Cost ($/MWh)	20.0	40.0	45.0	55.0

5. MARKET SIMULATIONS

One market design has been experimentally implemented in Matlab. The proposed joint energy-reserve market is a one-sided market with no demand-side participation. An Independent System Operator (ISO) deals with the security of the power grid and runs a central auction with price-inelastic load. Suppliers are allowed to submit separate offers for selling energy and spinning reserves. It is a two-product market, and separate nodal prices are set and paid for energy and reserves respectively. Suppliers take on the responsibility of determining their own tradeoff between the prices and quantities of energy and reserves in the offers they submit. The ISO will clear the market by doing a security-constrained optimization process. It is a single-settlement market-clearing mechanism, balancing the real-time market in which there is uncertainty about the actual pattern of loads and which one of the listed contingencies could occur. The optimization process consists of two stages:

1) A co-optimization is performed in stage one to minimize the expected costs of energy and reserves while meeting system load and transmission constraints, and maintaining certain grid security (cover listed credible contingencies). This stage determines the optimum patterns of energy dispatch and reserves.
2) Price-setting stage. Nodal energy and reserve prices are set in this stage. The payment will depend on whether the actual real-time system is in the base or in one of specified contingencies.

The solutions to the CO-OPT and the AOPF are solved by the Augmented Lagrangian Relaxation approach [7] using a commercial optimization package MINOS [8] interfaced into Matlab by C.E. Murillo-Sanchez [7]. We refer to this procedure as Responsive Reserves (RR).

The base load of the test market is set to be 220MW with 150.8 MW in area 1 and the rest in area 2, as shown in Figure 5. The load varies proportionally across the network from one trading period to another and is within $\pm 40MW$ of the base load. Most of the time (80%), the power grid runs smoothly without any failures, which is the designated base case. However, there is a 20% chance that one of the credible contingencies will occur. Six contingencies are considered in this test market, which include 10% unexpected load growth and the failure of the bigger unit (40 MW unit) of all firms except firm 2 (firm 1 and firm 2 are in similar situations, both of them affect the system in a similar fashion, hence only one of them is considered in the contingency list). The six contingencies will occur equally likely. Six firms, each manipulating two units, will submit energy and reserve offers to the market. Although the piecewise-linear offer curve can be decently handled(Appendix C), the offer curve is assumed to be linear here for the simple matter. Each unit is only allowed to submit one block and one offer price for energy and reserves respectively.

The first demonstration is the numerical check on the nodal energy and reserve prices by direct computation using (39) and (40). The calculated prices are pretty much consistent with those obtained by the perturbation procedures described in section 3.3. Selected sample results for generator buses are listed in Table 4.

Table 4. Nodal prices by perturbation vs. by direct computation

	BY PERTURBATION		BY DIRECT COMPUTATION	
Nodal price ($/MWh)	Energy	Reserve	Energy	Reserve
Bus 1	42.64	1.88	42.64	1.87
Bus 2	42.12	1.09	42.12	1.08
Bus 22	42.04	1.56	42.04	1.56
Bus 27	42.51	1.52	42.50	1.52
Bus 23	52.97	4.06	52.95	4.02
Bus 13	49.80	4.68	49.80	4.66

Table 5 shows an example market result. The system is in the base case with demand of 231 MW. All of the capacity is offered into the market, i.e., no withholding from the market. The unit number in the first column is used to label different generators such that unit 1 and 2 belong to firm1, unit 3 and 4 belong to firm 2, and so on. According to system load and offers, expensive units may be decommitted[2] (indicated by '0' in the second column of the table) from the market. In this case, unit 2 and 6 are not chosen for commitment. Basically, because of the transmission limits between two areas, there exists a zonal difference for both energy and reserve prices.

[2] Decommitment algorithm is described in Appendix D

Table 5. Example results of the proposed market for one trading period

UNIT	ON/OFF STATUS	ENERGY DISPATCH (MW)	ENERGY OFFER ($/MWH)	ENERGY PRICE ($/MWH)	RESERVE ALLOCATED (MW)	RESERVE OFFER ($/MWH)	RESERVE PRICE ($/MWH)
1	1	40.00	21.90	43.48	0.00	0.22	3.88
2	0	-	45.87	43.48	-	0.14	3.88
3	1	40.00	20.58	43.68	0.00	1.56	3.89
4	1	6.69	43.68	43.68	10.00	0.06	3.89
5	1	40.00	26.31	44.94	0.00	1.92	3.63
6	0	-	47.18	44.94	-	3.42	3.63
7	1	40.00	26.83	44.31	0.00	0.46	3.64
8	1	13.35	40.84	44.31	6.65	0.18	3.64
9	1	27.75	49.54	50.04	4.58	3.06	3.06
10	1	4.00	59.42	59.42	0.69	3.04	3.06
11	1	20.00	48.53	50.74	20.00	0.08	2.46
12	1	4.00	56.54	56.54	16.00	0.08	2.46

Further tests are performed on comparing the market performance between the proposed RR market and the standard practices used nowadays in the industry (i.e. specifying fixed amounts of reserves in different regions and minimizing the cost of meeting both load and the reserve requirements). This is actually one of the reasons that the RR market is of interest to us. The reserve requirement for the Fixed Reserve (FR) market is set such that the loss of the largest unit can be covered. Due to transmission limits between areas, the regional reserve requirement is forced: 40MW reserves are required inside area 2 and 60MW total are required for the whole system. So that, if the largest unit in area 2 (40MW) is lost, the 40MW reserve inside area 2 is able to cover the contingency; if the largest unit in area 1 (40MW) is lost, presumably, there will be 20MW reserve available in area 1 and another 20MW can be pulled out from the tie-lines (normally the power transferring from area 1 to area 2 congests the tie-lines) to handle the loss of the unit, and 20MW is also needed in area 2 to compensate the missing 20MW coming from the tie lines. The six contingencies for the RR market actually are selected such that both markets can cover the same set of contingencies in order to do fair comparisons (the contingency of unexpected 10% load growth is less severe than the unit-lost contingency, so the inclusion of it will not affect the fairness of the comparison).

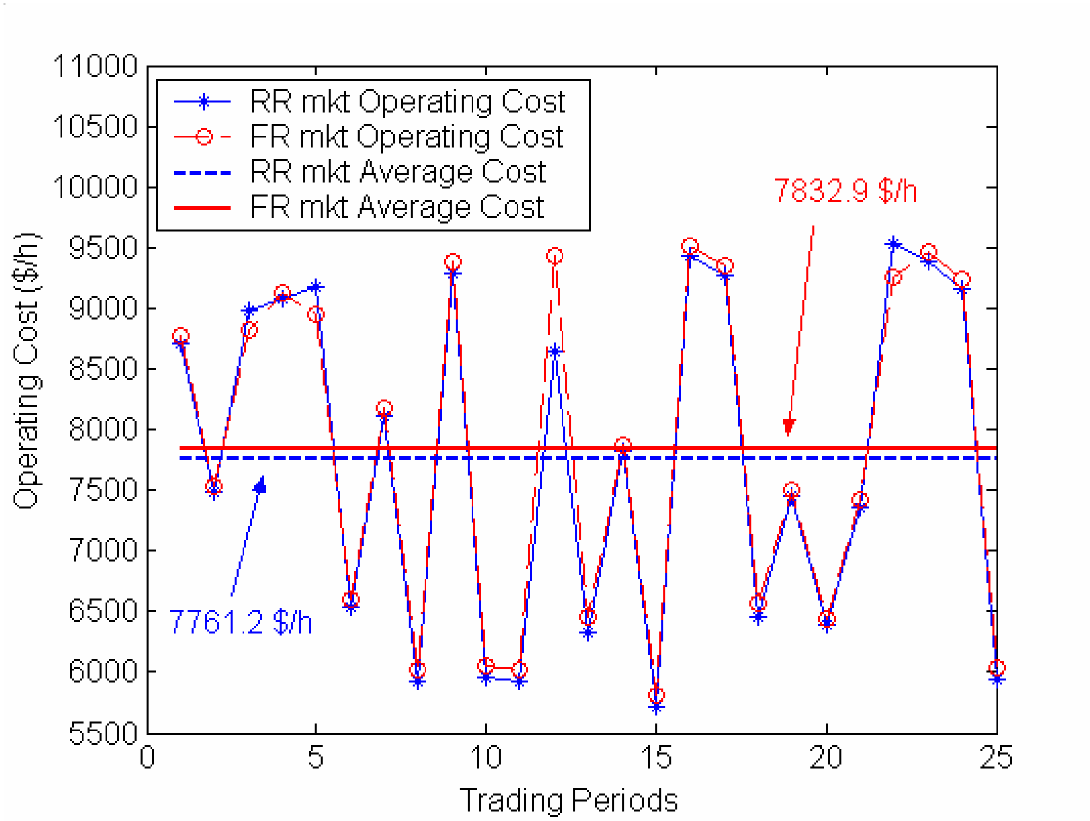


Figure 6. Comparison of operating costs between two markets (all marginal cost offers)

Figure 6 compares these two types of markets with all marginal cost offers ($4/MWh is taken as the marginal cost for reserves). The comparison is done over 25 trading periods with load variation and random contingency (in the contingency list) enforced. Most of the time, the operating cost for the FR market is higher than the RR market. But sometimes, the RR market does cost a little more. The average cost over 25 periods for the RR market is 7761.2$/h, which is slightly lower than that of the FR market, 7832.9$/h. The two markets have to meet the same amount of load, while the RR market has more constraints (extra constraints from contingency cases), usually the RR energy solution is a bit expensive than that of the less-constrained FR market. Hence, the reserve assignment gets credits for lowering the total operating cost for the RR market, that is, the "smart" reserve allocation reduces the amount of needed reserves. The statement is verified in Figure 7. The average amount (over 25 periods) of reserves required is 30.8MW in area 2 and 42.5MW in total, which explains the cost saving considering the requirement of 40MW in area 2 and 60 MW in total for the other market.

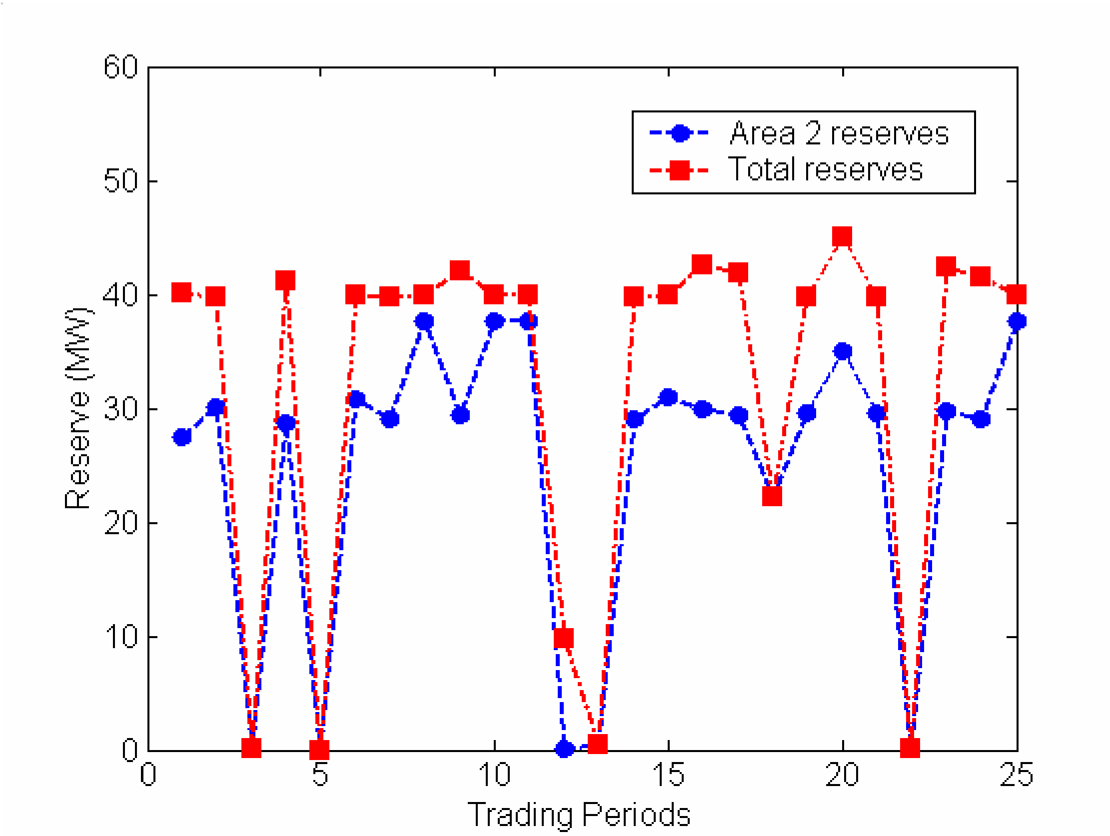


Figure 7. Reserves required for the RR market

Notice that area 2 actually is a load pocket, in which firm 5 and 6 possess market power, allowing them to manipulate the market. Figure 8 illustrates how the market manipulation affects the operating costs in both markets. The market outside the load pocket is still assumed to be competitive with everybody submitting marginal cost offers. While inside the load pocket, the two firms are putting very high offers, $90/MWh for both energy and reserves. The cost difference, 12098.0$/h in the FR market versus 10065.1$/h in the RR market, is big.

The cost saving is almost 20% in this case, which is a very impressive improvement. Again, the "smart" reserve allocation accounts for the big saving, which is shown in Figure 9. Figure 9 shows the energy dispatches and reserve allocations for each unit in both markets from one of the 25 trading periods. Clearly, in the FR market, due to the deterministic reserve requirement (40MW) inside the load pocket, although firm 5 and 6 make high offers, they can still sell reserves. However, for the RR market, the energy is dispatched so that the reserves are allocated only in the cheap area (area 1), avoiding high reserve charges in area 2.

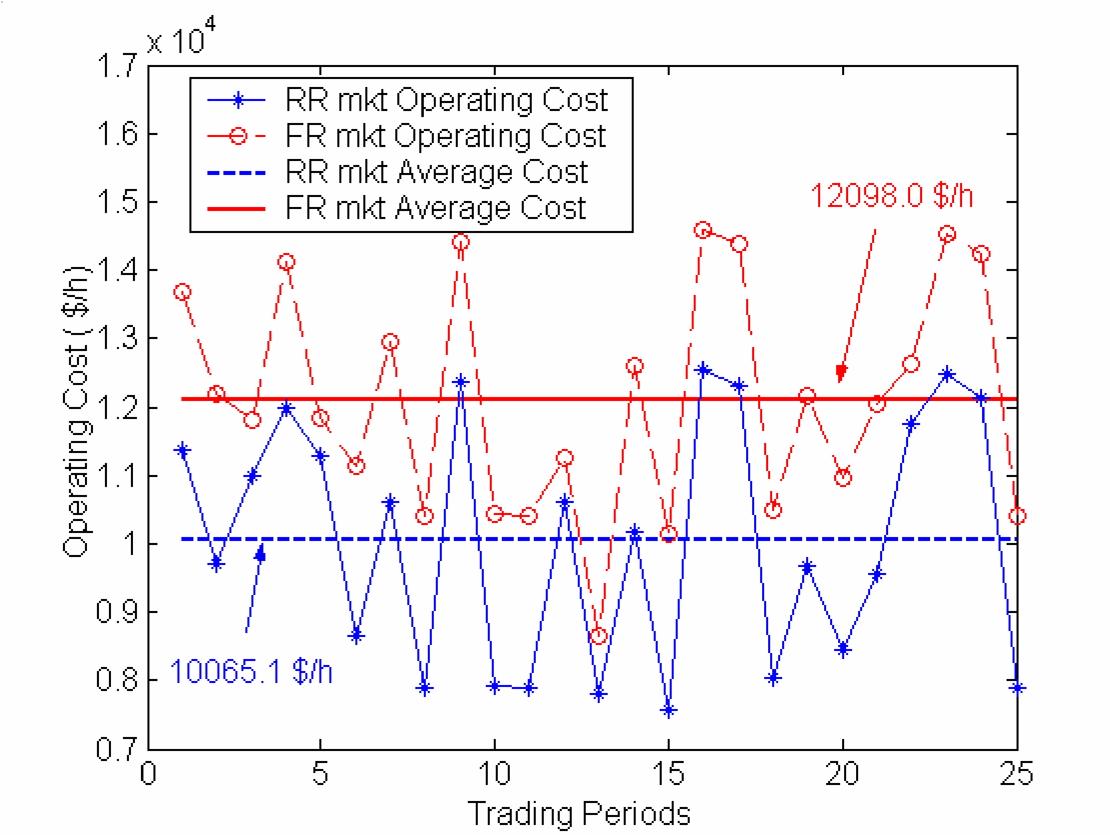


Figure 8. Comparison of operating costs between two markets (marginal cost offers for area 1 units, high offers for area 2 units)

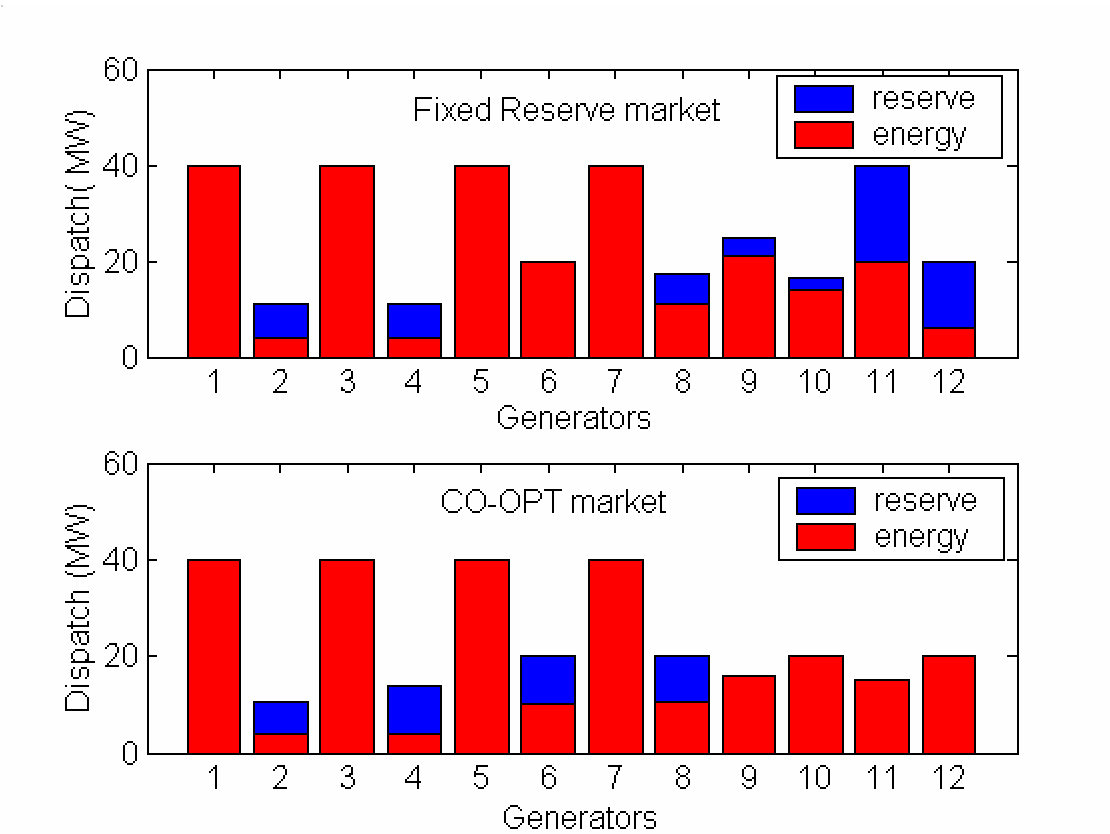


Figure 9. Comparison of reserve allocation patterns between two markets (marginal cost offers for area 1 units, high offers for area 2 units)

6. PRELIMINARY EXPERIMENTS

Experimental economics offers a tractable way to test the performance of complicated market structures. An important objective when running an experiment to test a market is to use knowledgeable subjects who can find and exploit flaws in the specified design. If the subjects represent suppliers, efficient markets should perform well and be robust to all of the attempts by experienced subjects to raise prices above competitive levels. Given these objectives, it is sensible to design a sequence of experiments using the same subjects to provide a pathway for acquiring experience about the characteristics of a complicated market. By starting with a relatively simple auction and adding new features one at a time, it is realistic to expect the subjects to learn how to exploit each new feature during the sequence of experiments. This type of evolution of a market structure in a sequence of experiments represents a close parallel to the way that a market like PJM has developed over time.

The following sequence of experiments were designed and tested by knowledgeable subjects (students who have experience with energy market experiments):

TEST I: **Fixed Reserves (Two Markets, "New York-like Market Rules")**
Specify fixed MW amounts of reserves in the load pocket and overall to meet a specified set of contingencies. Minimize the cost of energy and reserves.

TEST II: **Variable/Responsive Reserves (Two Markets, Co-optimization based)**
Minimize the expected cost of offers for energy and reserves over the same set of contingencies

TEST III: **Variable/Responsive Reserves (One Market + Opportunity Cost)**
Minimize the expected cost of offers for energy over the same set of contingencies. Pay reserves an opportunity cost for foregone "profits" on energy.

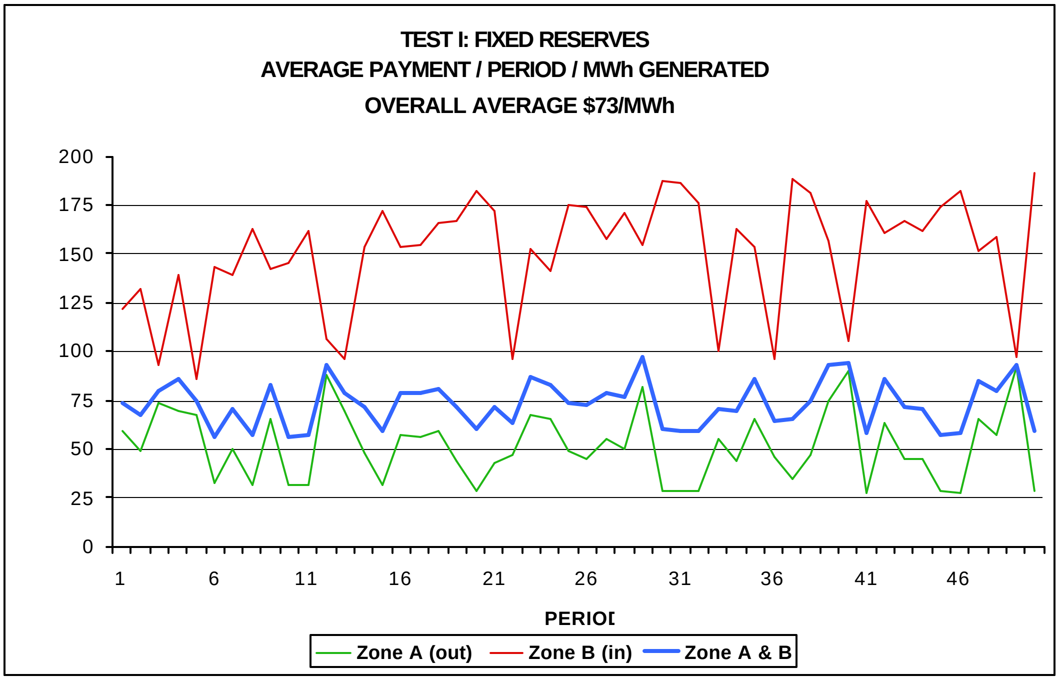
The objective is to determine,

1. Whether co-optimization of energy and reserves is more efficient than specifying fixed reserve requirements in different regions.
2. An efficient balance between paying reserves directly versus paying an opportunity cost for energy forgone.

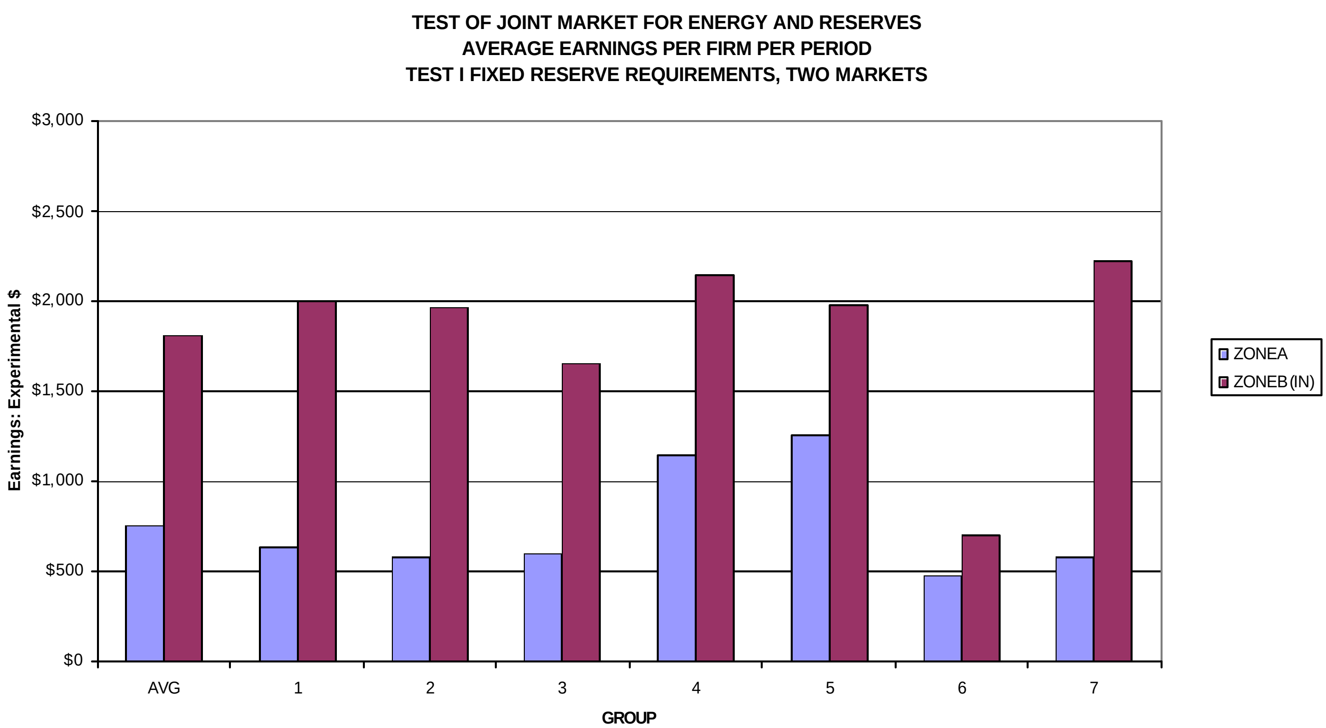
The basic market design[3] is to have ***two "firms" in a load pocket*** interact with competitive suppliers in other regions to provide energy and reserves for the whole network. These two generators are relatively expensive, and are represented by different individuals in a centralized auction. A price cap is enforced, but withholding capacity from the market is allowed. The other ***four generators outside the load pocket*** are relatively inexpensive and they are price takers. Computer agents represent these four generators and submit "honest offers" for all of their capacity at the true cost in all periods. ***Nodal prices*** for energy and reserves are determined using a uniform price auction in a centralized "smart" market with a full AC network. The system load on the network is ***price inelastic***, and it varies from period to period with no forecasting error. All of the markets will meet a specified level of reliability (i.e., providing reserves to cover the same set of contingencies). And the market is a one-settlement market.

There are seven groups of subjects participating in each of the test. Each test consists of 50 trading period. The experiment results for three tests are shown in Figure 10~12. The subfigures (a) show the average payment[4] of all units within zone A or zone B or both for each of the 50 trading period. The subfigures (b) show each group's average earnings of zone A units or zone B units, and also the overall average. The comparison of TEST I and TEST II results show that the overall average payment of the fixed reserve market (TEST I), $73/MWh, is significantly higher than the overall average payment for the responsive reserve market (TEST II), $62/MWh. Consequently, the average earnings from TEST I (around $1700) is more than that of TEST II (around $800). This suggests that the co-optimization is more efficient in revealing generators' true costs, which means the responsive market is more competitive. Notice that the market design includes a "load-pocket". In the fixed reserve market, only two firms compete against each other for zone B (the load pocket) reserve requirement; however, in the responsive reserve market, all six firms will compete for reserves because of the "smart" way of allocating reserves. So, the market power within zone B is easier to be discovered and exercised in the fixed reserve market, which partially accounts for the high prices in TEST I and also says that the responsive market is able to mitigate market power to some extent, if any. The result of TEST III is out of expectation. The average earnings of five groups are negative, only two groups can earn some small amount of money. The possible reason is that participants might not be aware of the color difference shown in the cumulative earnings (RED numbers mean debt while WHITE ones mean credit). They might think that they earn some money but in fact they owe money. But the energy offer prices in TEST III tend to be lower than those in TEST II, which is a good signal to the ultimate goal – competitive market.

[3] The detailed market set-up and experiment instructions are referred to Appendix E.
[4] Average payment = (total energy payout + total reserve payout)/total energy amount generated.

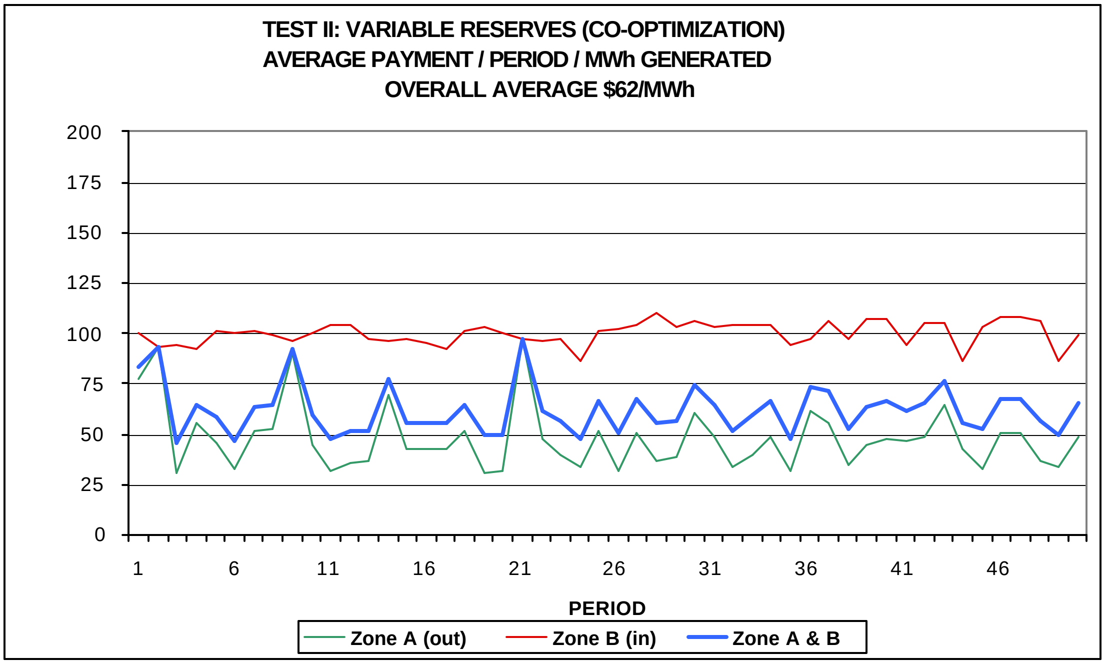


(a) Average payment

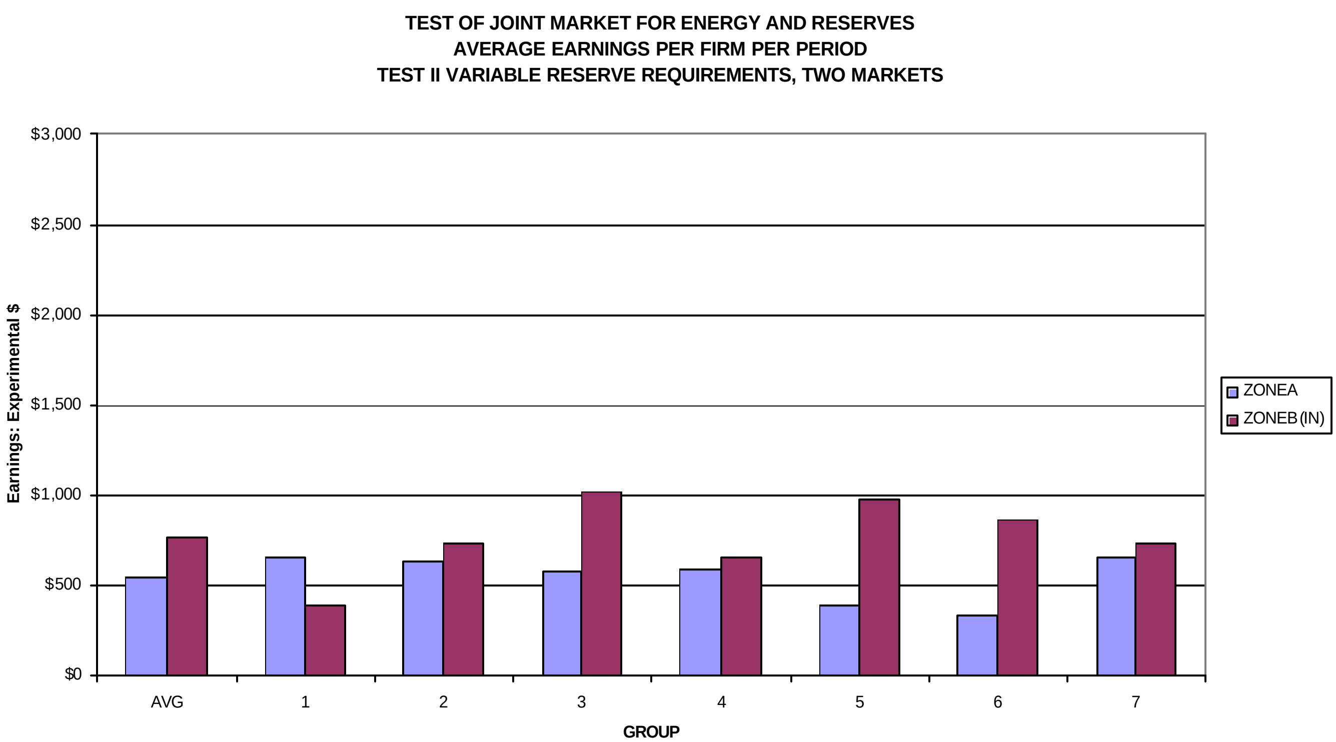


(b) Average earnings

Figure 10. Experiment results for TEST I

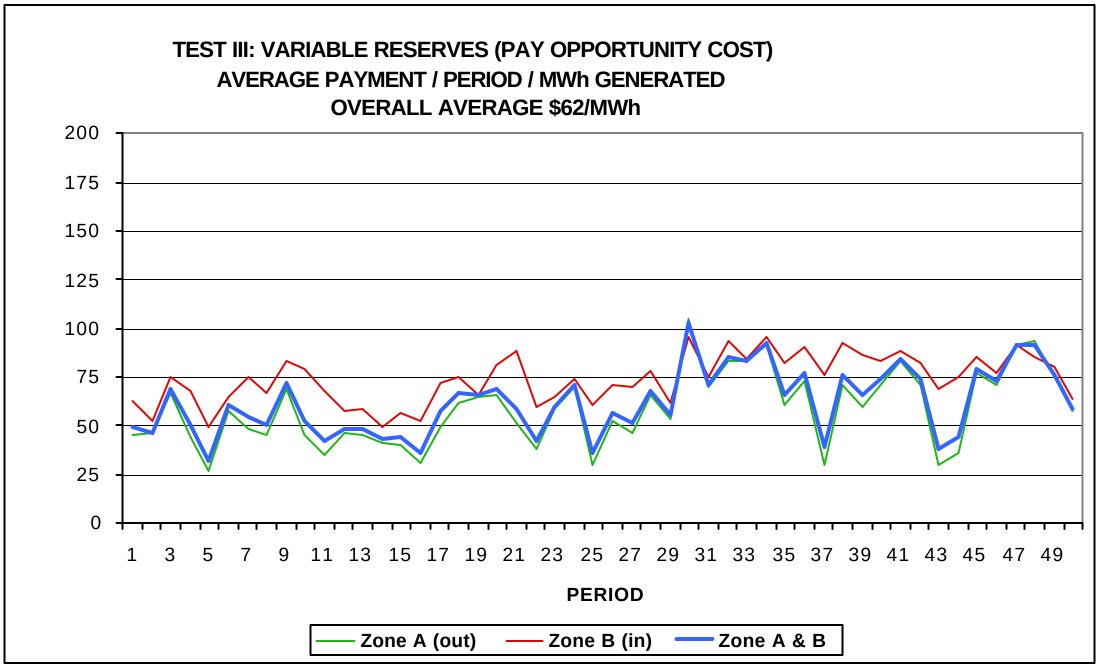


(a) Average payment

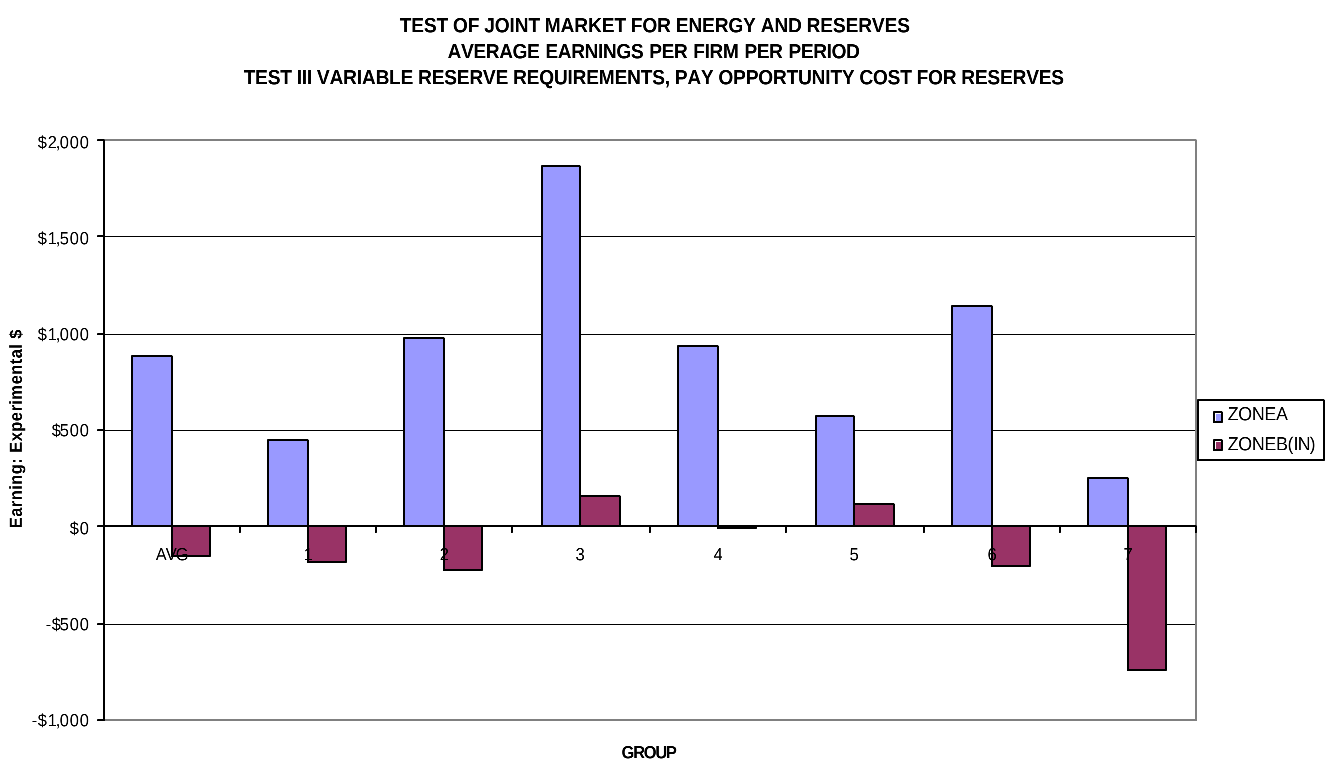


(b) Average earnings

Figure 11. Experiment results for TEST II



(a) Average payment



(b) Average earnings

Figure 12. Experiment results for TEST III

7. CONCLUSIONS AND FUTURE RESEARCH

The RR framework for an integrated energy-reserve market has been introduced in this work. The underlining optimization procedure provides not only locational assignments but also locational prices for both energy and reserves. Primary tests on the market design have been done based on a modified IEEE 30-bus system. The comparisons between the proposed RR market and the FR market in use show that the "smart way" of locationally assigning energy and reserves in the RR market requires less reserves to maintain the same level of system security as in the FR market, and therefore can improve the market performance – lower the operating cost. Energy and reserves interact more effectively with each other in the RR framework than they do in the FR market. Hence the RR market has the potential advantage of being more difficult to exploit when market power is a potential problem. In an FR market the demand for energy and reserves are both price inelastic. The test results do suggest,

- Using variable/responsive reserves (Co-optimization) is a promising way to reduce the market power that is inherent with fixed zonal reserves
- Paying reserves the opportunity cost of forgone profits for energy is a promising way to mitigate speculative behavior compared to paying separate prices for energy and reserves. (Greedy suppliers who try to get high energy prices also provide low cost reserves.)

However, these results are just preliminary, more tests with more experienced subjects are needed to produce more convincing results to support these observations.

The unit thermal constraints such as minimum up/down time and start-up costs are ignored for the current-stage development. And also the temporal issues are not yet honored in this work. However, the optimization framework and solutions are not necessarily limited by the assumptions made. Solving the unit commitment procedure based on the proposed optimization framework will be an important next step.

The test system used so far is only a small-size system. Applying the proposed RR framework to real-size systems is also an important next step. A combined economic and technical model for the New York State[5] has been developed and is ready for future test.

The RR framework is tested here in a one-settlement market set-up. However, the concept can also be applied to other market forms, for example, a two-settlement market. The co-optimization can be used, for example, in the day-ahead market to determine the optimum pattern of energy dispatch and reserves to meet the forecasted load and cover specified contingencies. In addition, various forms of day-ahead financial commitments, dependent upon different sets of market rules, can also be included in the co-optimization solutions. It is our intention that the RR framework will improve market performance and achieve better economic efficiency than the existing form of market with fixed requirements for reserves.

[5] see Appendix B

APPENDIX A: 30-bus Test System Data

This system is described in [10], but has been modified heavily. The data in the tables is almost exactly in *MatPower* format [11], which is in turn close to PTI format. The MVA base used is 100MVA.

Table A-1. Generator data for modified IEEE 30-bus system

Unit	Bus	P_G	Q_G	Q_{max}	Q_{min}	V_G	Base	Status	P_{max}	P_{min}
1	1	15.69	0	40	-10	1	100	1	40	8
2	1	7.85	0	20	-5	1	100	1	20	4
3	2	40.65	0	40	-10	1	100	1	40	8
4	2	20.32	0	20	-5	1	100	1	20	4
5	22	14.39	0	40	-10	1	100	1	40	8
6	22	7.2	0	20	-5	1	100	1	20	4
7	27	17.94	0	40	-10	1	100	1	40	8
8	27	8.97	0	20	-5	1	100	1	20	4
9	23	12.8	0	40	-10	1	100	1	40	8
10	13	6.4	0	20	-5	1	100	1	20	4
11	13	24.67	0	40	-10	1	100	1	40	8
12	23	12.33	0	20	-5	1	100	1	20	4

Table A-2. Bus data for modified IEEE 30-bus system

Bus	Type	P_D	Q_D	G_S	B_S	Area	V	q	baseKV	Loss Zone	V_{max}	V_{min}
1	3	0	0	0	0	1	1	0	135	1	1.05	0.95
2	2	6.71	2.95	0	0	1	1	0	135	1	1.1	0.95
3	1	29.68	11.16	0	0	1	1	0	135	1	1.05	0.95
4	1	11.75	1.86	0	0	1	1	0	135	1	1.05	0.95
5	1	0	0	0	0.19	1	1	0	135	1	1.05	0.95
6	1	0	0	0	0	1	1	0	135	1	1.05	0.95
7	2	35.25	12.67	0	0	1	1	0	135	1	1.05	0.95
8	1	18.55	13.95	0	0	1	1	0	135	1	1.05	0.95
9	1	0	0	0	0	1	1	0	135	1	1.05	0.95
10	1	8.43	2.32	0	0	3	1	0	135	1	1.05	0.95
11	1	0	0	0	0	1	1	0	135	1	1.05	0.95
12	1	3.7	1.74	0	0	2	1	0	135	1	1.05	0.95
13	2	0	0	0	0	2	1	0	135	1	1.1	0.95
14	1	2.05	0.37	0	0	2	1	0	135	1	1.05	0.95
15	2	11.5	2.47	0	0	2	1	0	135	1	1.05	0.95
16	1	1.16	0.42	0	0	2	1	0	135	1	1.05	0.95
17	1	2.97	1.35	0	0	2	1	0	135	1	1.05	0.95
18	1	4.49	0.89	0	0	2	1	0	135	1	1.05	0.95
19	1	13.33	3.36	0	0	2	1	0	135	1	1.05	0.95
20	1	3.09	0.69	0	0	2	1	0	135	1	1.05	0.95
21	1	4.24	2.17	0	0	3	1	0	135	1	1.05	0.95
22	2	0	0	0	0	3	1	0	135	1	1.1	0.95
23	2	26.94	9.49	0	0	2	1	0	135	1	1.1	0.95
24	1	12.64	7.79	0	0.04	3	1	0	135	1	1.05	0.95
25	1	0	0	0	0	3	1	0	135	1	1.05	0.95
26	1	5.09	2.67	0	0	3	1	0	135	1	1.05	0.95
27	2	0	0	0	0	3	1	0	135	1	1.1	0.95
28	1	0	0	0	0	1	1	0	135	1	1.05	0.95
29	1	3.49	1.05	0	0	3	1	0	135	1	1.05	0.95
30	2	15.4	2.21	0	0	3	1	0	135	1	1.05	0.95

Table A-3. Branch data for modified IEEE 30-bus system

From Bus	To Bus	R	X	B	MVA	Tap	Shift	Status
1	2	0.02	0.06	0.03	10000	0	0	1
1	3	0.05	0.19	0.02	10000	0	0	1
2	4	0.06	0.17	0.02	10000	0	0	1
3	4	0.01	0.04	0	10000	0	0	1
2	5	0.05	0.2	0.02	10000	0	0	1
2	6	0.06	0.18	0.02	10000	0	0	1
4	6	0.01	0.04	0	10000	0	0	1
5	7	0.05	0.12	0.01	10000	0	0	1
6	7	0.03	0.08	0.01	10000	0	0	1
6	8	0.01	0.04	0	10000	0	0	1
6	9	0	0.21	0	10000	0	0	1
6	10	0	0.56	0	10000	0	0	1
9	11	0	0.21	0	10000	0	0	1
9	10	0	0.11	0	10000	0	0	1
4	12	0	0.26	0	8	0	0	1
12	13	0	0.14	0	10000	0	0	1
12	14	0.12	0.26	0	10000	0	0	1
12	15	0.07	0.13	0	10000	0	0	1
12	16	0.09	0.2	0	10000	0	0	1
14	15	0.22	0.2	0	10000	0	0	1
16	17	0.08	0.19	0	10000	0	0	1
15	18	0.11	0.22	0	10000	0	0	1
18	19	0.06	0.13	0	10000	0	0	1
19	20	0.03	0.07	0	10000	0	0	1
10	21	0.03	0.07	0	10000	0	0	1
10	22	0.07	0.15	0	10000	0	0	1
21	22	0.01	0.02	0	10000	0	0	1
15	23	0.1	0.2	0	10000	0	0	1
22	24	0.12	0.18	0	10000	0	0	1
23	24	0.13	0.27	0	15	0	0	1
24	25	0.19	0.33	0	10000	0	0	1
25	26	0.25	0.38	0	10000	0	0	1
25	27	0.11	0.21	0	10000	0	0	1
28	27	0	0.4	0	10000	0	0	1
27	29	0.22	0.42	0	10000	0	0	1
27	30	0.32	0.6	0	10000	0	0	1
29	30	0.24	0.45	0	10000	0	0	1
8	28	0.06	0.2	0.02	10000	0	0	1
6	28	0.02	0.06	0.01	10000	0	0	1

APPENDIX B: A Combined Economic And Technical Model For New York State

In order to test concepts associated with the co-optimized energy and reserves a real-system-size test system was constructed based on data that was received from the *New York Power Pool* (NYPP) in 1999. The NYPP is the predecessor to the NYISO. If the concept works on this model we feel we can update the data at a later date to better reflect current operating and economic conditions. What makes this collection of data unusual is that it contains both technical and economic information based on actual planning and operating conditions so that unit commitment, economic dispatch and security constraints can be investigated in the context of the combined energy/reserve markets we propose to test. Economic information for loads and generators as well as the system load case information was provided by the NYPP. Modifications to the data files were made in order to make their formats suitable for both the MatPower™ [11] and PowerWorld™[12] power system simulation packages. Although the PowerWorld simulator is not suitable for this study because we do not have access to the source code, its visual interface provides a way to visualize the system and to provide a check on the final results. We use MatPower, a Matlab-based optimal power flow program developed at Cornell because we can modify the code to suit our formulation of the problem.

One of the problems faced in power systems studies is data management. A typical power system data file contains information about the physical and operational capabilities of lines, buses, transformers, shunt components, and generators. Other system data files may include economic information about generators, zones, and areas. The type and format of information in an input data file depends on the study that is going to be performed, and the power system simulation package (algorithm) that is going to be employed. Economic information about loads and generators consist of energy blocks, and energy incremental costs used to define their bid sizes for a specified hour, and date. It also includes the expected energy capacity of the suppliers over that period. With this information, piecewise cost models were constructed, and a MatPower™ input file format was created. Several problems were encountered while treating the data in the files. For instance, economic information was not provided for all the generators and on occasion the economic information provided lacks consistency. These problems were identified, and modifications were made so that the information could be used in MatPower™.

The data provided by NYPP consisted of 24 load cases in PTI (*.raw) standard format, and the economic information for suppliers and customers' energy block bids for a day in periods of an hour. A file containing a single field description for the generation units in the system accompanies this information. This last file is used to identify the generators in the economic data file. The PTI format does not allow for the inclusion of economic information about the generators. For this reason one-line diagrams were created in PowerWorld for each of the load cases in order to incorporate the economic data. PowerWorld features direct access to the system information through its one-line diagram and graphical representations of the simulations. Also provides a useful tool for performing studies on system violations on bus voltage magnitudes and in line flows. Combining the load case information with the economical information for customers and suppliers, a MatPower format input file is created that will allow using MatPower's OPF algorithm. The problems encountered with the data are covered next.

Standard data file formats like PTI (*.raw) and IEEE (*.cf) can be loaded into PowerWorld. The one-line diagram is not contained in those formats. Therefore it has to be drawn manually over the system data. The one-line diagrams were created form a one-line diagram provided by NYPP. This diagram contains line numbers, bus nominal KVs, and names for substations or neighborhoods. But it does not contain bus numbers. PowerWorld identifies its buses by their numbers or names. This proved to be a significant obstacle to completing the diagram.

To number the buses in the PowerWorld one-line diagram the following procedure was implemented:

- Relate the name, and nominal KV of a bus in the data file with the nominal KV and substation name in the NYPP diagram. Substations may have a group of buses represented in the one-line diagram, and sometimes two or more buses may have the same nominal voltage. At this stage a group of buses are located on the diagram.
- Perform the same procedure with surrounding substations sharing lines with the previous substation.
- From the group of bus numbers previously located, identify the line and transformer information fields in the case data, and match the numbers for the buses that are interconnecting substations.
- The remaining buses in a substation may be generation buses, load buses, or transformer buses (isolated). The generation buses are identified by using the generator fields. The load buses are identified by using the bus field. Special transformers (LTC, phase shifters) have two sets of fields in the data. One set is within the line and transformers fields and another set that refers to their special settings—control bus number, upper and lower limit of turn ratio and phase shift, upper and lower limit of controlled volts, turns ratio step increment and

transformer impedance correction. Common transformer buses have to be identified from the special transformer buses by using both fields. Transformer fields include two extra-fields (transformer off nominal turn's ratio, and transformer phase shift angle).

- Sometimes generators or loads are not shown in the diagram. Generators and loads are attached to substations by transformers. By identifying those transformers the generators, and loads bus numbers can be located.

From this procedure it is clear that if the diagram does not have enough details (many of the buses not shown) the puzzle is extremely difficult to solve. The data file contained information for 6 areas, including the NYPP area. The one-line diagram provided by NYPP only had explicit information for the NYPP (bus and substation names). Therefore, the only area that appears in the PowerWorld one-line representation is the NYPP. Nevertheless, the underlying information corresponds to the entire case. Figure B-1 shows an example of the visual output from PowerWorld for the data set.

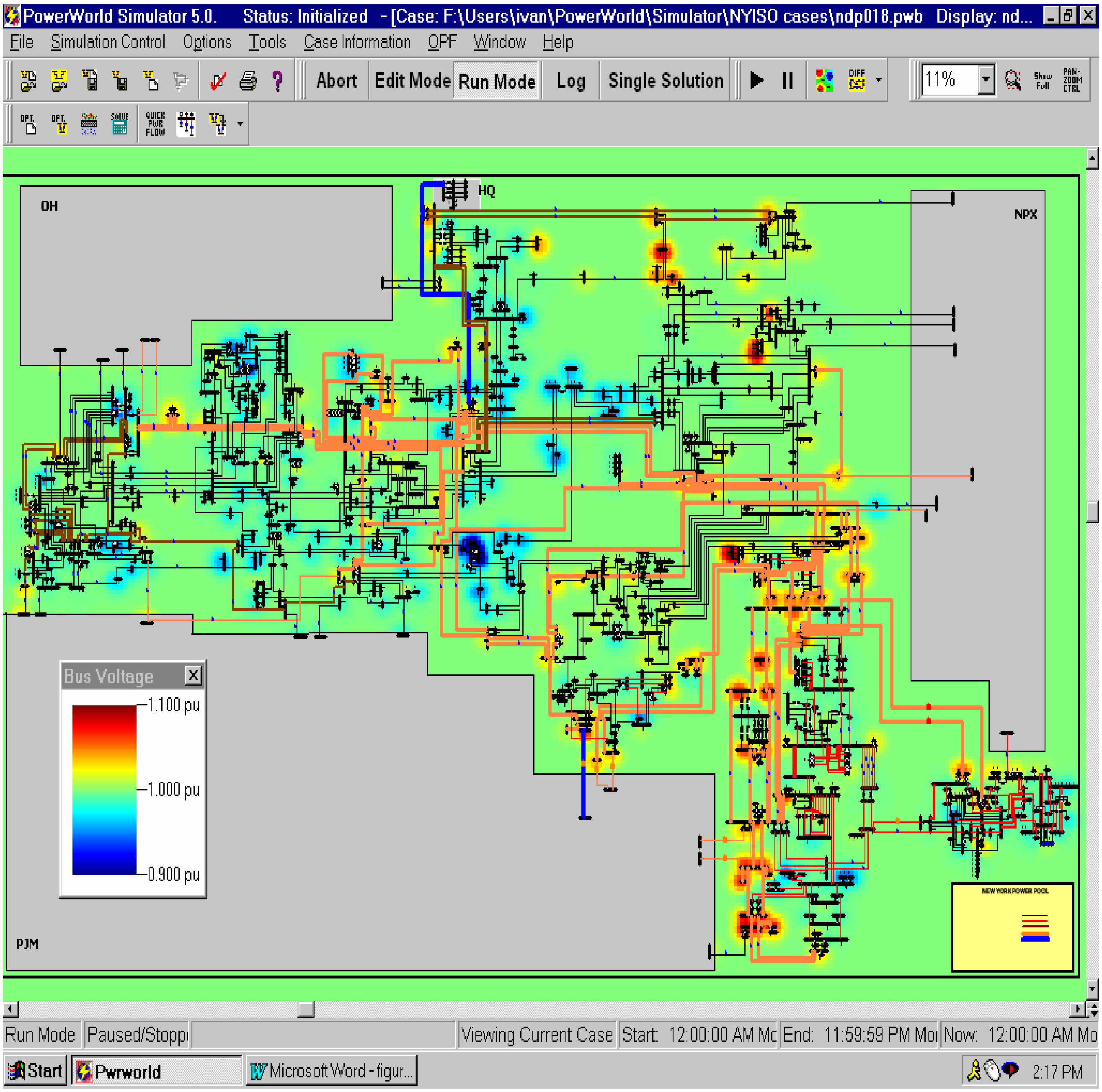


Figure B-1. PowerWorld visual with voltage magnitude contouring : ndp018.pse

APPENDIX C: Handling Piecewise Linear Energy/Reserve Cost Curves

The energy/reserve offers made in the wholesale market are often piecewise linear cost curves as shown in Figure 2 of section 3.1, which can be transformed into equivalent costs with linear restrictions for convenient implementation.

Generally, if the original problem is

$$\min\{\ f(x)\ |\ \ g(x)=0\} \tag{C-1}$$

with $f(x)$ piecewise linear, convex, and specified by breakpoints $\{(x_k, f_k),\ \ k=0,\cdots N\}$, then one can denote the cost value by y, use adjacent breakpoints to impose linear restrictions on y and minimize y:

$$\min \quad y \tag{C-2}$$

such that

$$g(x)=0 \tag{C-3}$$

$$y \ge \frac{f_{k+1}-f_k}{x_{k+1}-x_k}(x-x_k)+f_k, \quad k=0,\cdots N-1 \tag{C-4}$$

in effect constructing a convex basin for y based on the linear segments of $f(x)$.

Two possible energy/reserve offer patterns are considered for the transformation.

Case A.

The offer pattern A is shown in Figure C-1. Both energy and reserve offers are made independently, although the starting point of reserve blocks is not fixed, which depends on actual energy dispatch. They both are fixed block bids with price and quantity specified for each segment. The capacity limit, $P_{\max}$, and the ramping limit, $R_{\max}$ (in MW for 10 minutes), are specified (the actual quantity of available reserve is the $\min\{R_{\max}, P_{\max} - P_{dispatch}\}$) for energy offer and reserve offer respectively.

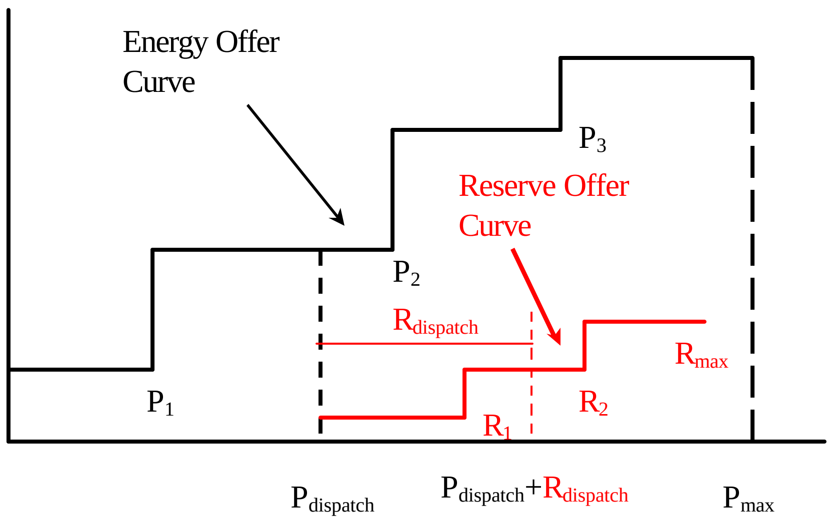


Figure C-1. Incremental cost curves (pattern A) for energy and reserves

The original problem

$$\min \; f_p(P) + f_R(R) \qquad \text{s.t.} \quad \{\text{SYSTEM CONSTRAINTS}\} \tag{C-5}$$

then can be transformed into

$$\min \quad y_p + y_R \tag{C-6}$$

s.t. {SYSTEM CONSTRAINTS}, and

$$y_p \ge \frac{f_p^{k+1} - f_p^k}{P_{k+1} - P_k}(P - P_k) + f_p^k, \quad k = 0, \cdots N-1 \tag{C-7}$$

$$y_R \ge \frac{f_R^{l+1} - f_R^l}{R_{l+1} - R_l}(R - R_l) + f_R^l, \quad l = 0, \cdots M-1 \tag{C-8}$$

Case B.

The offer pattern B is shown in Figure C-2. Both energy and reserve quantity offers have the same number of blocks and the same breakpoints, but their price offers are independent of each other. The capacity limit, $P_{\max}$, and the ramping limit, $R_{\max}$ (in MW for 10 minutes), are also specified (the actual quantity of available reserve is the $\min\{R_{\max}, P_{\max} - P_{dispatch}\}$) for energy offer and reserve offer respectively.

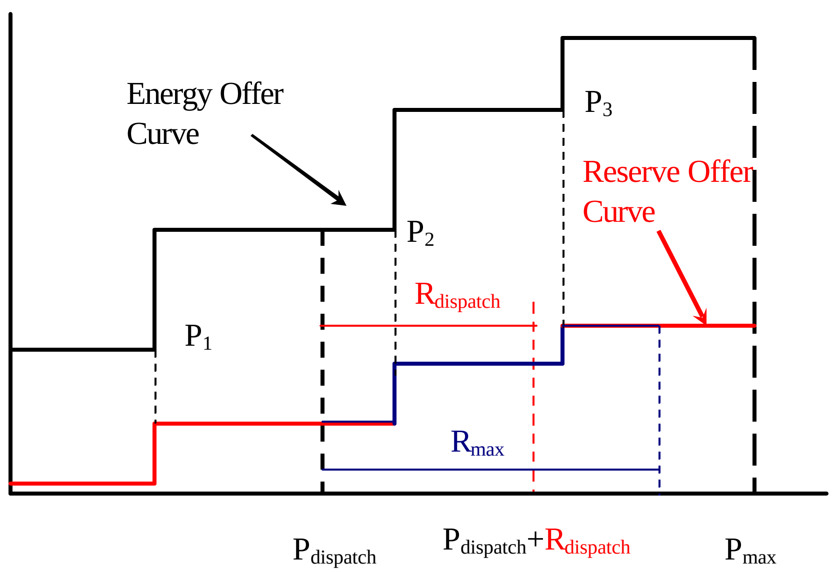


Figure C-2. Incremental cost curves (pattern B) for energy and reserves

In this case, the blue curve (part of the red curve) is the actual incremental cost curve for reserves. One problem with the reserve cost calculation is that the actual breakpoints ($P_k - P_{dispatch}$, $k \in \{k \mid P_k \geq P_{dispatch}, k = 0, \cdots N\}$) change over iterations. Hence, keeping the linearity of transformation as (C-8) for reserves is not possible. One way to solve this problem is to transform the multiple-block offers of one generator into multiple one-block offers of multiple generators (each generator offers one block). Take Figure C-2 for example, the 4-block offers from one generator then can be replaced by 4 one-block offers from 4 generators. Generally, suppose one generator's offer (dispatch denoted as (P_g, R_g)) has N blocks with breakpoints specified by $\{P_k, k = 0, \cdots N\}$, a N-generator (dispatch denoted as $(\overline{P}_g{}^k, \overline{R}_g{}^k), k = 1, \cdots, N$) mathematical equivalent can be made. The problem (C-5) then can be transformed into

$$\min \quad y_{P_g} + y_{R_g} \tag{C-9}$$

s.t. { SYSTEM CONSTRAINTS}, and

$$P_g = \sum_{k=1}^{N} \overline{P}_g^{\,k} \tag{C-10}$$

$$R_g = \sum_{k=1}^{N} \overline{R}_g^{\,k} \tag{C-11}$$

$$\overline{P}_g^{\,k} + \overline{R}_g^{\,k} \le P_k - P_{k-1}, \quad k = 1, \cdots N \tag{C-12}$$

$$\overline{P}_g^{\,k} \le P_k - P_{k-1}, \quad k = 1, \cdots N \tag{C-13}$$

$$y_{P_g} = \sum_{k=1}^{N} y_{\overline{P}_g^{\,k}} \tag{C-14}$$

$$y_{R_g} = \sum_{k=1}^{N} y_{\overline{R}_g^{\,k}} \tag{C-15}$$

$$y_{\overline{P}_g^{\,k}} = \frac{f_p^k - f_p^{k-1}}{P_k - P_{k-1}} \times \overline{P}_g^{\,k}, \quad k = 1, \cdots N \tag{C-16}$$

$$y_{\overline{R}_g^{\,k}} = \frac{f_R^k - f_R^{k-1}}{P_k - P_{k-1}} \times \overline{R}_g^{\,k}, \quad k = 1, \cdots N \tag{C-17}$$

APPENDIX D: Unit Decommitment

The CO-OPT formulation described in the previous section has no mechanism for completely shutting down generators which are very expensive to operate. Instead they are simply dispatched at their minimum generation limits. A simple unit decommitment algorithm is used here to be able to shut down these expensive units.

The decommitment process starts off with the system commitment initialized to all available units. To reach an economic operation, some expensive units will be shut down *one at a time* with the most costly unit (evaluated by the decommitment criteria) decommitted first until no further reductions in total cost are possible. The algorithm is actually based on the ***Dynamic Programming*** technique but without complete enumeration, only a subset is evaluated.

The algorithm proceeds as follows:

Step 1. *Initial commitment S is set to include all units.*

Step 2. *Solve a CO-OPT with commitment S, optimum cost =* f_0.

Step 3. *Find out all generators (denoted as* $G_1 \sim G_m$*) dispatched at their minimum generation limits with commitment S. If empty, go to step 7, else go to step 4.*

Step 4. *for k = 1:m*

Solve a CO-OPT with commitment S and G_k *being decommitted. Record the optimum cost* f_k.

Step 5. $f_i \leftarrow \min\{f_k, k=1, \ldots, m\}$

Step 6. *If* $f_i < f_0$

Then $f_i \rightarrow f_0$

S + decommitting $G_i \rightarrow S$

Go to step 3.

Else

Go to step 7.

Step 7. *Stop, S is the final commitment.*

APPENDIX E: Testing Joint Markets for Electricity and Reserves (Instruction)

1. Introduction

This test is an example of the economics of decision making for supplying electricity using PowerWeb. You represent an experienced trader in selling electricity to meet load (demand) in a central auction run by an Independent System Operator (ISO). The design of this new test is relatively complicated because it is based on a “smart” market that incorporates the physical characteristics of the electrical grid, including transmission line congestion, transmission losses and location-specific pricing. In addition, unexpected failures of generators and transmission lines (contingencies) can occur. To keep the system running in a contingency, additional generating capacity (reserve capacity) is purchased in advance to replace equipment that fails. Consequently, generating capacity can be used to meet load or act as reserves and there are separate markets for each. You as a trader can sell your capacity in both markets. It is very important to read over the instructions thoroughly.

Two different tests will be conducted corresponding to two different ways of specifying the requirements for reserves. Each test will be run for 20 trading periods. The instructions below describe the both parts of the test. You will receive additional instructions before the second part begins. Throughout the experiment, the decisions you make determine your earnings, and you will be paid in cash in proportion to your earnings. However, since there are important locational differences for generators on the network, individual traders do not have the same exchange rate. Your objective is to maximize your earnings over the series of 40 periods. Please do not communicate with any of the other participants during the experiment.

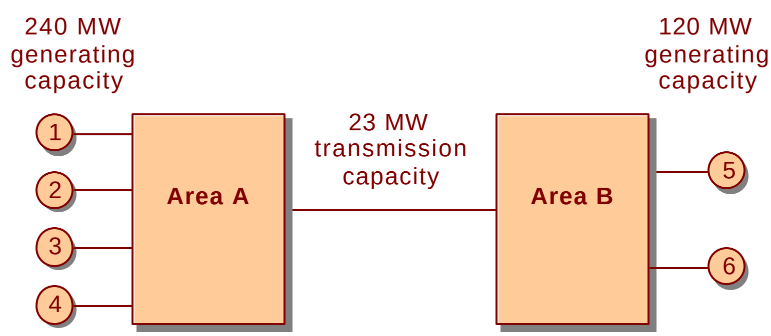


Figure E-1. Transmission Network Block Diagram

2. General Market Characteristics

You operate one of six firms in an electricity market run by an ISO. Firms 1, 2, 3, and 4 are located in Area A while Firms 5 and 6 are located in Area B (see *Figure E-1*). The transmission capacity between Area A and Area B is relatively limited compared to the transmission capacity within the two areas. Each firm owns two electric power generators. (Firm 1 controls Gen 1 and Gen 2. Firm 2 controls Gen 3 and Gen 4, etc.) with a combined maximum capacity of 60 MW. The first generator has a maximum capacity of 40 MW for generating electricity, and the second has a maximum capacity of 20 MW. (Since there are physical limits on how quickly a generator can respond to a failure on the system, the maximum capacities for reserves are lower than the capacities for electricity.) Your generators are both at the same location on the transmission network. The load, on the other hand, is dispersed at various locations throughout the network.

Each firm sells capacity into two closely related markets: electricity and reserves. The ISO purchases electricity and reserves to minimize costs under the specified rules of the market. The two tests correspond to two different ways of selecting reserves. In each case, the rules are designed to ensure that the system will continue to operate over a specified set of contingencies. When capacity is purchased to generate electricity, generation costs are incurred (e.g. for fuel). Capacity purchased for reserves does not incur any generation costs. Most of the time (80%), the system runs smoothly with no failures, but there is a 20% chance that one of the contingencies will occur in any given trading period. For example, a contingency may involve the failure of one of your generators. When a contingency occurs, some of the capacity scheduled as reserves is used to generate electricity instead. However, the market results that you see will always reflect the actual operating conditions.

Since the contingencies occur at random, your generators may fail more than once, while another firm has no failures. Since prices are likely to be higher in contingencies, because fewer resources are available to meet load, your earnings will be adversely affected when one of your generators has an unexpected failure. To avoid serious inequities in earnings among the traders, each firm has an insurance policy that pays a fixed amount per MW whenever one of your generators fails. Therefore, the insurance policy reduces the disadvantages for firms that experience many random failures.

The software used by the ISO in this smart market is relatively sophisticated and involves solving a large number of highly non-linear equations. It is possible that the algorithm will get stuck and not converge to the optimum solution in an acceptable time. Under these circumstances, the auction for that period will be cancelled and no payments will be made. The test will move immediately to the next period.

2.1. Submitting Offers into the Auction

In each trading period, you will see the *Offer Submission* page shown in *Figure E-2*. On the left-hand side, the **forecasted load (MW)**, the **total installed capacity on the system (360 MW),** and the **price cap ($100/MW)** are shown in the *system data* table. The forecasted load varies from one period to the next and is within ± 40 MW of 220 MW (this value is highlighted with a yellow background). For both tests, the actual load is equal to the forecasted load under normal operating conditions (i.e., there is no forecasting error, but load may be affected in some contingencies). The total installed capacity and the price cap are constant for both tests. The price cap is the maximum price you can offer for selling electricity or reserves.

On the right-hand side of the page, the *generator data* table displays the following information for your two generators (the values contained in *Figure E-2* correspond to Firm 6, operating Generators 11 and 12, and these values will differ from your information in the actual test):

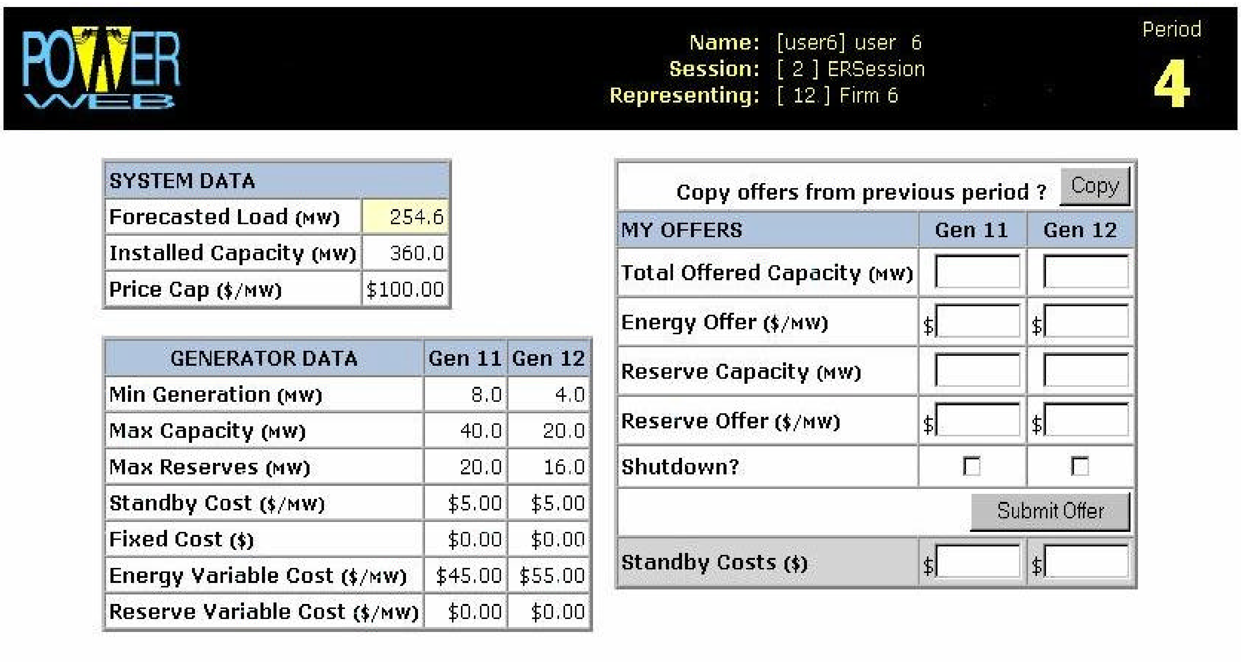


Figure E-2. Offer Submission page

Min Output (MW) – the minimum amount of capacity you can offer to sell for electricity. Due to physical limitations on the generator, it is not possible to operate the generator at less than this output level without shutting it down completely.

Max Output (MW) – the maximum amount of capacity you can offer to sell for electricity and reserves combined.

Max Reserves (MW) – the maximum amount of capacity that can be sold as reserves, determined by the physical ability to respond to a contingency (the ramp rate).

Standby Cost ($/MW) – the opportunity cost for capacity offered into the auction, regardless of how much you sell.

Fixed Cost ($) – a single payment to cover capital costs in each trading period, regardless of how much you sell.

Energy Variable Cost ($/MW) – the generation cost per MW for all capacity sold as electricity.

Reserve Variable Cost ($/MW) – the reserve cost is zero.

All capacity limits and costs remain constant throughout both tests, but they do differ across firms. Offers that violate limits will not be accepted and an error message (in red) will appear above the *my offers* table.

You submit offers to sell electricity and reserves by making entries in the *my offers* table. In this table, the engineering term **energy** is used for **electricity**. Specifically, you make the following decisions for each of your generators:

Total Offered Capacity (MW) – the maximum amount of capacity you wish to sell as either energy or reserves.

Energy Offer ($/MW) – the price you wish to sell capacity for energy.

Offered Reserve Capacity (MW) – the maximum amount of the total offered capacity that you wish to sell as reserves (Note: Offered Reserve Capacity < Max Reserves AND < (Max Output – Min Output)).

Reserve Offer ($/MW) – the price you wish to sell capacity for reserves.

Shutdown? – checking this option means you forego the opportunity to sell capacity from the generator but avoid paying the standby costs (offers can not be entered when shutdown is checked for a generator).

Standby Costs ($) – are calculated and displayed automatically when you enter a value for the Total Offered Capacity.

Submit Offer – This button sends the offers to the ISO.

Copy – This button will set the same offers as the last period, and then individual entries can be edited.

The total standby costs for each generator are calculated and displayed automatically at the bottom of the table when you enter the Total Offered Capacity. After all of the capacity and price offers have been entered, hit the **Submit Offer** button to send the offers to the ISO. Be sure to check your offers BEFORE hitting the Submit Offer button because you **CANNOT** recall the offers once they have been submitted.

3. The Auction

When the ISO receives energy and reserve offers, a **last accepted offer auction** is performed to minimize costs and determine how much capacity to purchase as energy and as reserves from each generator and what prices to pay. This auction is first described in a simplified context without the effects of the transmission network, and then the full implementation used in the tests with the network included is described.

3.1. Simplified Auction (*with no network*)

When there are no network effects, all firms receive the same market-clearing price for energy and a second market-clearing price for reserves. This corresponds to a straightforward extension of a uniform price auction to two products.

How does the ISO choose capacity for energy and reserves?

Using optimization techniques, the ISO simultaneously chooses the lowest energy offers to meet load and the lowest reserve offers to meet reserve requirements.

How are prices determined?

The ISO ranks all accepted energy (reserve) offers from lowest to highest. The market-clearing price – paid to all accepted offers – for energy (reserves) is equal to the price of the last accepted (highest priced) offer. **(Note: the Min Generation constraint on capacity implies that it is possible to make an offer below the market-clearing price and still not get selected by the ISO.)**

3.2. Complete Auction (*with a network*)

For both of the tests that you will conduct, the generators and loads are connected by a transmission network, and operations are always constrained to obey the laws of physics governing the flow of electricity. The operation of the network is also constrained by the physical limitations and the location of the equipment used to generate and transmit electricity. The complications associated with a transmission network introduce two phenomena that affect the auction results in a smart market.

Transmission Losses

A small percentage of the energy produced by the generators is dissipated by the transmission lines. The amount lost depends on the flows in the lines and the length of the lines, among other things. **Transmission losses imply that the total amount of electricity purchased is greater than the total load.** The amount of transmission losses is dependent on operating conditions (losses tend to be higher when limits on transmission capacity are reached).

Congestion

There are limits on the amount of electricity that can be transmitted from one location to another. Some of the limits are simple capacity limits on lines (e.g., only 23 MW can transfer between Area A and Area B), but others are more subtle system limits with complex engineering explanations. Congestion occurs when one or more of these network limits is binding. **Congestion implies that some inexpensive generation may be unusable due to its location, making it necessary to utilize higher-priced capacity in a different location.**

The effects of transmission losses and congestion are handled in a smart market by adjusting all prices paid to reflect the true value of generation (or reserves) at each location. In your tests, **each generator receives prices that are specific to its location (nodal pricing).** Hence, even though the most expensive source in a region sets the price for all purchases, the prices actually paid are not strictly uniform.

How does the ISO choose capacity for energy and reserves?

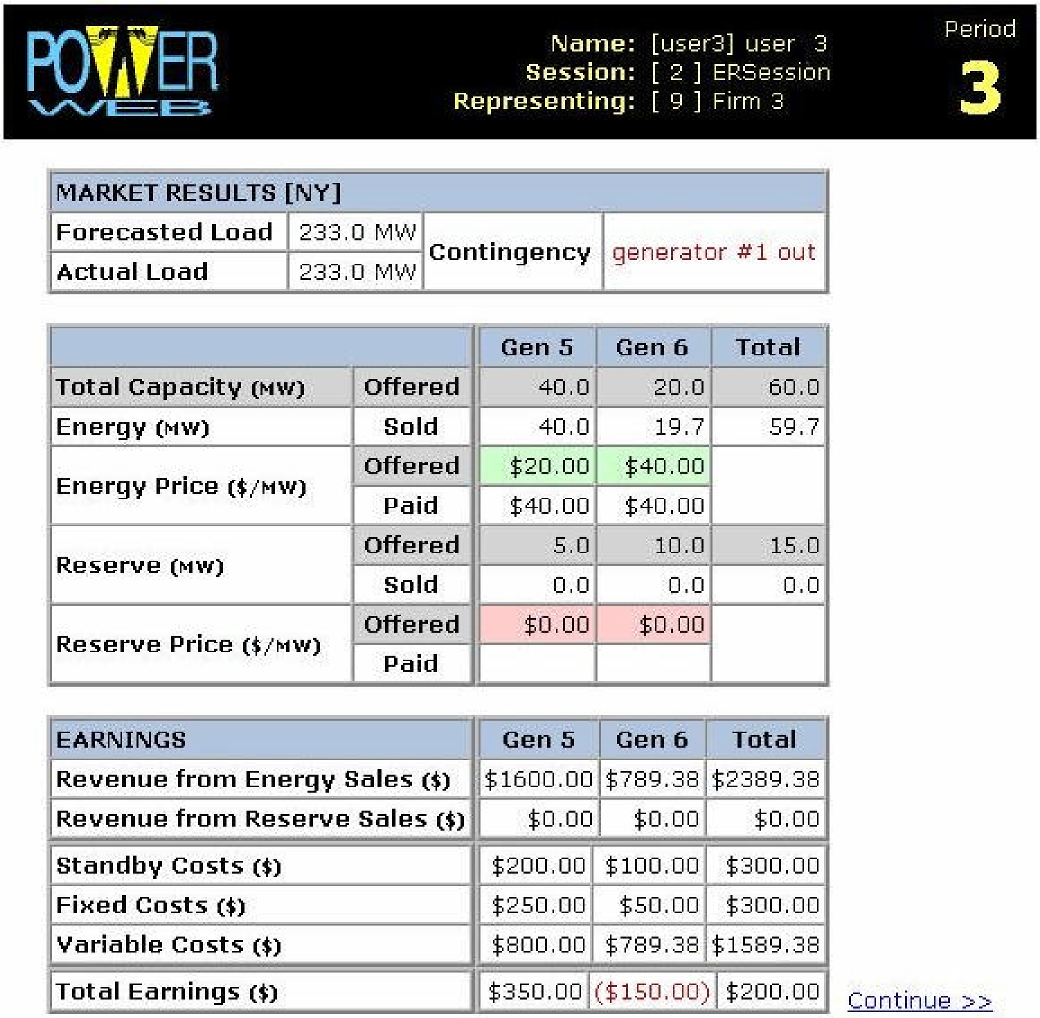
A complex non-linear optimization program selects energy and reserve offers to minimize costs subject to meeting the load and the reserve requirements, as well as satisfying the physical constraints of the transmission system.

How are prices determined?

All price offers are effectively adjusted by a location-specific transmission factor. The ISO ranks all the adjusted energy (reserve) offers from lowest to highest. The market-clearing price for energy (reserves) is set by the last accepted offer (adjusted for location). The price paid for each MW of energy (reserves) incorporates a transmission adjustment for the location. When transmission constraints are binding, the market may effectively split into separate regions with different generators setting the prices in different regions.

4. Reviewing the Auction Results

After submitting your offers, please wait patiently until others finish submitting their offers and the PowerWeb completes all of the calculations. The results appear automatically as soon as they are available. The *Auction Results* page, shown in *Figure E-3*, displays two tables.



POWER WEB

Name: [user3] user 3
Session: [2] ERSession
Representing: [9] Firm 3

Period
3

MARKET RESULTS [NY]			
Forecasted Load	233.0 MW	Contingency	generator #1 out
Actual Load	233.0 MW		

		Gen 5	Gen 6	Total
Total Capacity (MW)	Offered	40.0	20.0	60.0
Energy (MW)	Sold	40.0	19.7	59.7
Energy Price ($/MW)	Offered	$20.00	$40.00	
	Paid	$40.00	$40.00	
Reserve (MW)	Offered	5.0	10.0	15.0
	Sold	0.0	0.0	0.0
Reserve Price ($/MW)	Offered	$0.00	$0.00	
	Paid			

EARNINGS	Gen 5	Gen 6	Total
Revenue from Energy Sales ($)	$1600.00	$789.38	$2389.38
Revenue from Reserve Sales ($)	$0.00	$0.00	$0.00
Standby Costs ($)	$200.00	$100.00	$300.00
Fixed Costs ($)	$250.00	$50.00	$300.00
Variable Costs ($)	$800.00	$789.38	$1589.38
Total Earnings ($)	$350.00	($150.00)	$200.00

Continue >>

Figure E-3. Auction Results page

The *cleared offer* table displays your energy and reserve offers along with the total capacity actually sold as energy and reserves, and the corresponding prices paid (adjusted for transmission effects). The top table shows the **Forecasted Load**, the **Actual Load** and whether or not a

Contingency occurred. The middle table shows the **Total Capacity Offered** with a GREY background. (The whole column is GREY if the generator is withheld, and DARK RED if the generator fails in a contingency.) Offered prices have a GREEN background if they were accepted and a RED background if they were rejected by the ISO. Prices paid are given only if some capacity is sold.

The capacity of energy and reserves sold and the corresponding prices paid are used to determine net earnings in the bottom table. The *earnings* table displays your revenues from the sale of energy and reserves, along with all the costs you incurred (including the fixed cost and standby costs). The earnings for a generator are:

earnings = (energy sold) x (nodal price) + (reserves sold) x (nodal price) - (energy sold) x (operating cost) – (total offered capacity x standby cost) - (fixed cost)

MARKET HISTORY													
Period		System Load (MW)	Generation (MW)		Market Share	Reserves (MW)		Energy Price ($/MW)		Avg Market Price ($/MW)	Reserve Price ($/MW)		Earnings ($)
			Gen 5	Gen 6		Gen 5	Gen 6	Gen 5	Gen 6		Gen 5	Gen 6	
3	In	233.0	40.0	20.0	26%	5.0	10.0	$20.00	$40.00		$0.00	$0.00	
	Out	233.0	40.0	19.7	26%	0.0	0.0	$40.00	$40.00	$53.36			$200.00
2	In	221.5	40.0	20.0	27%	5.0	10.0	$20.00	$40.00		$0.00	$0.00	
	Out	221.5	40.0	10.0	23%	0.0	10.0	$42.64	$42.64	$49.71		$2.64	$358.57
1	In	247.1	40.0	20.0	24%	5.0	10.0	$20.00	$40.00		$0.00	$0.00	
	Out	247.1	40.0	12.8	21%	0.0	6.1	$40.00	$40.00	$53.15		$0.00	$200.01
1 - 3		Cumulative Earnings:											$758.58
		Cumulative Earnings * Exchange Rate (1/1000):											$0.76

Figure E-4. Market History

The market history table in *Figure E-4* summarizes the results for all trading periods (the most recent period is at the top). For each trading period, the first line summarizes the inputs (GREY background, except for offered prices which are GREEN if accepted and RED if rejected). The second line summarizes the quantities sold and prices paid. If a contingency occurs, then **System Load** and **Earnings** have a DARK RED background. In addition, generators that fail in a contingency are also colored DARK RED. At the bottom of the table, the **Cumulative Earnings** and the conversion to real dollars paid are given for all trading periods. When you have finished reviewing the auction results for a trading period, hit the Continue button on the right to move on to the next trading period.

Specific Characteristics for Three Tests

Test I: Fixed Reserve Requirements

Firms submit offers to sell energy and reserves in a central auction run by an ISO, who selects the least expensive combination of offers to meet the system load AND meet reserve requirements. **In this experiment, the reserve requirements are specified as physical constraints. In each trading period, the ISO must purchase 60 MW of reserves, with at least 40 MW of this total in Area B. Actual purchases of reserves may be lower if contingencies occur.** Nodal prices are paid for all accepted energy and reserve offers. A situation may arise where the amount of capacity offered for energy (reserves) is not adequate to meet load (reserve requirements). In this case, expensive generators from outside your market provide the additional energy (reserves). However the market clearing price that you receive for energy (reserves) will never be higher than the price cap ($100/MW).

Test II: Responsive Reserve Requirements

Firms submit offers to sell energy and reserves in a central auction run by an ISO, who selects the least expensive combination of offers to meet the system load AND meet reserve requirements. **In this experiment, the reserve requirements are determined by a co-optimization that ensures that the system can still operate under all of the specified contingencies (same ones as Test I). The amount of reserves purchased is not constant and depends on the offers submitted.** Nodal prices are paid for all accepted energy and reserve offers. A situation may arise where the amount of capacity offered for energy (reserves) is not adequate to meet load (reserve requirements). In this case, expensive generators from outside your market provide the additional energy (reserves). However the market clearing price that you receive for energy (reserves) will never be higher than the price cap ($100/MW).

Test III: Pay the Opportunity Cost for Reserves

Firms submit offers to sell **energy only** in a central auction run by an ISO, who selects the least expensive combination of offers to meet the system load AND meet reserve requirements. **Capacity submitted to the auction can be used for energy or for reserves (up to the limit of the ramp rate). Capacity selected for reserves is paid the opportunity cost of the foregone energy (nodal price for energy – offer for energy). The rationale for this method of payment is that the "profit/MW" will be the same regardless of whether a generator's capacity is used for energy or for reserves. However, this means that submitting a high offer may increase the price paid for energy but it will lower the price paid for reserves.** In this experiment, the reserve requirements are determined by a co-optimization that ensures that the system can still operate under all of the specified contingencies (same ones as Test II). The amount of reserves purchased is not constant and depends on the offers submitted. Nodal prices are paid for all accepted energy and reserve offers. A situation may arise where the amount of capacity offered for energy (reserves) is not adequate to meet load (reserve requirements). In this case, expensive generators from outside your market provide the additional energy (reserves). However the market clearing price that you receive for energy (reserves) will never be higher than the price cap ($100/MW).

REFERENCES

[1] A.A. Fouad, "Dynamic Security Assessment Practices in North America", *IEEE Transactions on Power System*, v 3, n 3, Aug, 1988, p 1310-1321

[2] R. Rajaraman, L. Kirsch, F.L. Alvarado, and C. Clark, "Optimal Self-Commitment Under Uncertain Energy and Reserve Prices", book chapter in *The Next Generation of Electric Power Unit Commitment Models*, International Series in Operations Research & Management Science, vol.36, Kluwer Academic Publishers, Boston, April, 2001

[3] Y.Chen, and V. Venkatasubramanian, " Automatic On-line Controller for Coordinated Slow Voltage Control", Accepted by IEEE Trans. on Power Systems, (to appear)

[4] F.L. Alvarado, et.al, "The Value of Transmission Security," EPRI TR-103634 Research Project 4000-14, Final Report, Aug., 1994

[5] A. J. Wood, and B.F. Wollenberg, *Power Generation, Operation, and Control*, John Wiley & Sons, Inc., New York, 1996.

[6] S. Stoft, *Power System Economics: Designing Markets for Electricity*, Wiley-IEEE Press, May, 2002

[7] C. E. Murillo-Sanchez, R.J. Thomas, "Thermal Unit Commitment With A Nonlinear AC Power Flow Network Model", book chapter in *The Next Generation of Electric Power Unit Commitment Models*, International Series in Operations Research & Management Science, vol.36, Kluwer Academic Publishers, Boston, April, 2001

[8] B.A. Murtagh, and M.A. Saunders, *MINOS 5.5 User's Guide*, Standford Univ. Systems Optimization Laboratory Technical Report SOL 83-20R

[9] F.C. Schweppe, M.C. Caraminis, R.D., Tabors, and R.E. Bohn, *Spot Pricing of Electricity*, Kluwer Academic Publishers, 1988

[10] O. Alsac, and B. Stott, "Optimal Load flow with Steady-State Security", *IEEE Trans. On Power Apparatus and Systems*, Vol. PAS 93, No. 3, 1974, pp745-751

[11] R.D. Zimmerman, and D. Gan, "*MATPOWER: A Matlab Power System Simulation Package User's Manual*", [online]:http://www.pserc.cornell.edu/matpower/matpower.html

[12] PowerWorld Simulator, [online]: www.powerworld.com/products/simulator.html

TECHNICAL PAPERS

J. Chen, J.S. Thorp, R.J. Thomas, and T.D. Mount, "Locational Pricing and Scheduling for an Integrated Energy-Reserve Market", 36th Hawaii International Conference on System Science, (HICSS-36), Hawaii, Jan. 2003